Public and Private Equity in a General Equilibrium Model of Occupational Choice

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Preliminary and Incomplete

Abstract

Recent empirical evidence has documented that over the past decade the total value of the public equity traded in the US had risen significantly compared to the total value of private equity, while their relative returns have not changed much. These trends would be hard to reconcile within a standard representative agent portfolio choice model. This paper develops a general equilibrium occupational choice model which can account for the observed dynamics of relative quantities and prices of public and private equity. It argues that the relative returns to two assets have changed little because an increase in the available quantity of public equity was accompanied by an increase in the demand for public equity driven by the changes in the occupational structure of the population and rising wealth inequality. Both of these effects are derived in the model endogenously and caused by the improvements in the technologies allowing private firms to go public.

Keywords: initial public offering, public and private equity, occupational choice, wealth distribution

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1 Introduction

In a recent empirical study Moskowitz and Vissing-Jorgensen (2002) have documented a number of interesting facts about the characteristics of private equity using the Survey of Consumer Finances data. Even though their main finding is concerned with the relative risk and return offered by private and public equity,[1] another interesting observation emerges from their results. It turns out that over the past decade the total value of public equity in the US had risen enormously compared to the total value of private equity (the ratio of the values of private to public equity was 1.08 in 1989, and gradually fell to 0.39 by 1998).

A representative agent portfolio choice model would suggest that such dramatic change in the relative quantities of traded assets should be accompanied by a considerable change in their relative returns (e.g. the returns of public equity should go up in order to explain why consumers' investment in this type of asset had risen so much).[2] However, the data does not support this prediction: the relative returns of public and private equity have not changed much over the past decade (their ratio had remained very close to 1 over all this period, though both return rates have gone up). Hence, an alternative explanation is needed to understand the dynamics of relative quantities and returns of public and private equity. This paper is an attempt to provide one.

We develop a general equilibrium model which explicitly describes the link between private and public equity by introducing the possibility of Initial Public Offerings (IPOs): entrepreneurs start their private firms, eventually some of them may go public thereby generating an inflow into public equity, which is available for everyone's investment. The rate of return to public equity, as well as the price of a private firm at IPO are determined endogenously, which in turn affects the returns to entrepreneurial investment in their firms. We use this model to argue that the above mentioned facts about the relative quantities and returns of public and private equity can be explained by the recent changes in the capital markets, which made

[1] The authors document that even though private equity investment is very risky, private equity does not seem to pay any premium over public equity, which became known in the literature as the 'Private equity premium puzzle'.

[2] Moskowitz and Vissing-Jorgensen (2002) do not report whether there was any substantial change in the riskiness of private or public equity returns. I should look at the data myself.

it easier for entrepreneurs to sell their firms through IPOs and lead to the changes in the occupational structure of the population and in the wealth distribution.

The key features of our model are the presence of borrowing constraints, scarcity of entrepreneurial ideas and barriers to going public. In fact, we assume exogenous arrival rates for entrepreneurial ideas and for IPO opportunities and analyze the effects of an increase in the latter. Intuitively, if more entrepreneurs can sell their firms through IPOs, the amount of public equity offered on the stock market rises and the amount of privately owned firms declines (and, hence, the ratio of the values of private to public equity drops). If there were no shift in the demand for public equity, such change would imply that the price of the publicly traded assets must fall (the rate of return rise).

However, we argue that as more firms are sold through IPOs, the demand for public equity might also rise for two reasons. First, after an entrepreneur sells his firm through IPO, he gives up his ownership shares and has no access to private equity investment any more until he comes up with a new business idea and creates a new privately owned firm. Hence, as more entrepreneurs go public and the total number of entrepreneurs falls – the total number of workers (who can invest only in public equity) rises. Second, selling one's company through IPO generates significant capital gains. Thus, if it becomes easier to go public, more very rich agents appear. These agents tend to save a lot, which drives the demand for public equity even further up. Consequently, an increase in the supply of public equity happens simultaneously with an upward shift in the demand for it; therefore, its price (return) might behave very differently from what might be suggested by a representative agent portfolio choice model.

Why do we believe that such mechanism is a natural explanation of the facts documented by Moskowitz and Vissing-Jorgensen (2002)? First, due to the nature of the Survey of Consumer Finances data, the investment in private equity analyzed in their paper is in fact, as the authors themselves acknowledge, investment made by entrepreneurs into their own businesses (86% of all private equity in SCF is held by consumers with active management role in the company where they invest in private equity). Thus it seems natural to connect private equity investment with entrepreneurial activity. Second, it has been well documented that the frequency of IPOs has increased a lot over the past 15 years (e.g., Jovanovic and Rousseau

2001). A part of this rise in IPO rate is attributed to the dot com boom, but other factors, such as easier access of venture capitalists, better information technologies, etc., undoubtedly play an important role too. Thus we analyze the effects of an exogenous increase in the IPO rate in our model.

The main driving forces of the mechanism described in this paper are the changes in the occupational composition of the population and the wealth distribution. Both of them find support in the data. The data from the Survey of Consumer Finances indicates that the number of entrepreneurs measured as a fraction of the total population has gone down from 14% in 1989 to 7.8% in 1998 (e.g. Quadrini 2000). Simultaneously, rising wealth inequality has been discussed a lot in the recent literature, with capital gains being usually listed among its main reasons (e.g., Smith 1999).

To our knowledge, this is the first paper that attempts to endogenize the link between privately owned firms and public equity in a general equilibrium model in order to derive some predictions about their relative returns. A number of recent papers have modelled the decisions of entrepreneurs to go public. For example, Chari et. al. (2006) study the welfare effects of capital gains taxes in a partial equilibrium framework (with exogenously given rate of return to public equity). In some sense, we extend a simplified version of their model to a general equilibrium environment. Bhattacharya and Ravikumar (2001) and Clementi (2002) analyze at which stage of firm development an entrepreneur would choose to sell his firm through IPO and study how this decision can be affected by the development of the capital markets and reducing credit market imperfections. The predictions of our model regarding the timing of IPO are more stylized; our entrepreneurs choose to go public whenever they can. Even though such prediction finds some support in the data (Darby and Zucker 2002), it is still very extreme and additional features could be added to our model in order to improve its performance along this dimension.

The paper is organized as follows. Section 2 develops the model. Section 3 discusses the main properties of the equilibrium allocation. Section 4 describes our preliminary numerical results. Section 5 outlines the major shortcomings of the predictions of the current model and discusses potential directions for its improvement.

2 The Model

We set up an infinite-horizon discrete-time model populated by a continuum of infinitively-lived agents of measure one. Every agent maximizes the present value of expected life-time utility $E_0 \sum_{t=0}^{+\infty} \beta^t u(c_t)$, where $\{c_t\}_{t=0}^{+\infty}$ is the stream of the agent's consumption (possibly uncertain), $\beta \in (0, 1)$ is the time discount factor and E_0 is the expectation operator (conditional on the state of the world in period 0). The instantaneous utility function $u(c)$ is strictly increasing, strictly concave and bounded from above. There is no disutility from leisure, so all the workers work full time; assume that each one of them is endowed with 1 unit of time in every period. The consumption feasibility set as well as the stochastic properties of the agents' income process depend on the occupation of the agent (a worker or an entrepreneur) and will be discussed later.

We also assume that capital markets are incomplete and borrowing is not allowed. Agents can save in a risk-free asset (*public equity*) which pays off interest rate r. Entrepreneurs can also invest in their own firm (which we will associate with *private equity* investment). We describe workers' and entrepreneurs' decision problems in details below.

2.1 Workers

In the beginning of every period a worker has some wealth $a \geq 0$ (non-negative since borrowing is not allowed) and receives wage w in return for his full-time labor services (the same for all workers, i.e. we assume that there is no heterogeneity in workers' skills) The worker decides in every period how much of his total assets to consume and how much to save for the future. In the following period the worker with probability η will get 'hit with an idea' and will be able to start his own business if he wants to. For simplicity we will assume that ideas are one-time opportunities: if the worker decides not to start the business, his idea will be lost and he will need to wait for another idea to arrive before he would have a chance to open up a business again. Thus the decision problem of the worker can be written down as

$$V_w(a) = \max_{a' \geq 0} \{u(a + w - \frac{a'}{1+r}) + \\ + \beta(1-\eta)V_w(a') + \beta\eta \cdot \max\{V_w(a'), V_e(a')\}\}, \tag{1}$$

where $V_e(a')$ is the value of a business owner with wealth a' defined in the next section. Lets' denote the decision rules associated with problem (1) by $a'(a)$ (optimal investment in public equity by the worker with current wealth a) and $I_{we}(a)$ (the decision of whether or not to open up a business if he has an idea). Obviously, $I_{we}(a) = 1$ if $V_e(a) \geq V_w(a)$ and $I_{we}(a) = 0$ otherwise.

2.2 Private equity: entrepreneurs

We will distinguish between two types of business owners: (i) those who own the business in the beginning of the period and can still decide whether to remain in its possession in the current period or give up their property rights by either going public or by simply closing their business down; (ii) those who have already decided to manage (and own) their firm in the current period. For brevity, we will refer to these two types as 'owners' and ''entrepreneurs' respectively and denote their values by $V_o(a)$ and $V_e(a)$.

Let's describe entrepreneurs' decision problem first. As well as workers, entrepreneurs are credit constrained. They can invest in the public equity and in their own business (private equity). Their business investment is risky: the amount of capital k invested in business today pays off $F(k, l)$ in the following period if the business survives and brings zeros payoff if the firm closes down. Thus for the business owners investing in public equity serves as a partial insurance against loosing all their savings. Exit occurs with (exogenously given) probability x, and if it happens, an entrepreneur has no other choice but become a worker.

The payoff function $F(k, l)$ depends on the firm's capital stock k and the number of workers l employed in his firm. As it is common in occupational choice models, we assume that $F(k, l)$ has decreasing returns to scale, so that entrepreneurial profit can compensate for giving up positive wage w. Apart from potential differences in wealth, we assume that entrepreneurs are all identical (i.e. they have the same production technology $F(k,l)$). Potentially, this assumption could be relaxed while doing calibration exercise; but we try to keep the model as simple as possible at the current stage in order to better understand the underlying mechanisms.

The decision problem of the entrepreneur can be described by the following dynamic programming problem:

$$\begin{aligned} V_e(a) &= \max_{b,k,l\geq 0}\{u(a-wl-b-k)+\beta(1-x)V_o(a_h')+\beta x\cdot V_w((1+r)b)\}, \\ &\text{s.t.} \quad a_h' = (1+r)b + F(k,l). \end{aligned} \tag{2}$$

Let's denote by $b(a)$, $k(a)$ and $l(a)$ are optimal levels of savings in public equity, capital investment and labor input derived from this problem.

In the beginning of the next period, if the firm survives, the owner will have three options: to continue managing his firm, to close it down and become a worker or to go public. Suppose the business can be sold through IPO at a price Π (which is endogenized in the next section; for now it is only important to point out that Π does not depend on the wealth level of the entrepreneur). By selling his business, the agent gives up his entrepreneurial idea and has to become a worker. Thus the value of going public for the incumbent with current wealth a is given by $V_w(a+\Pi)$. We assume that the opportunity to go public arrives with exogenous probability p (the same for all business owners). This parameter can be related to the level of development of capital markets, availability os venture capitalists facilitating IPOs, etc. Thus the decision problem of the owner can be written down as

$$\begin{aligned} V_o(a) = (1-p)\cdot &\max\{V_e(a), V_w(a)\}+ \\ &+p\cdot \max\{V_e(a), V_w(a), V_w(a+\Pi)\} \end{aligned}$$

Obviously, $V_w(a)$ is monotone in wealth and thus $V_w(a)$ disappears from the last max operator:

$$V_o(a) = (1-p)\cdot\max\{V_e(a), V_w(a)\} + p\cdot\max\{V_e(a), V_w(a+\Pi)\} \tag{3}$$

The policy associated with the first max operator in (3) was already defined when we introduced the optimal rules for the worker's problem (1). The last max operator appearing in the right hand side of (3) determines the optimal choice of whether or not two go public if such opportunity is available: $I_{IPO}(a) = 1$ if $V_e(a) \leq V_w(a + \Pi)$ and $I_{IPO}(a) = 0$ otherwise.

All the agents solve their decision problems given the parameters of the model (preferences $u(c)$ and β, technology $F(k, l)$, and the probabilities η of coming up with an idea, x of loosing the business and p of being able to go public) as well as the prices w, r and Π determined endogenously. We are going to focus on the stationary equilibrium allocation in which prices remain constant; that is why the state space in the problems (1)-(3) includes only individual asset level. The next section describes the structure of the public firms and the determination of the firms' value at IPO Π.

Finally, note that return to private equity (entrepreneurial activity) consists of two components: the regular business payoff generated by $F(k, l)$ and the proceedings from IPO. We will describe the details of computing the average return to private equity while describing the numerical exercise in the last section. But it is obvious that, other things constant, increases in Π or in p should raise the return to private equity.

2.3 Public firms

We assume that IPO market is perfectly competitive, so the price of a business Π at the moment of IPO is equal to the expected discounted flow of profits which the firm's shareholders receive after it becomes public.[3]

The difference between public and private (entrepreneur-owned) firms is twofold. First, public firms have excess to external financing, i.e. they can always borrow enough funds at a competitive interest rate r to finance their capital investment and wage payment. Second, we assume that public shareholders perfectly diversify their ownership shares and thus they discount the sum of the dividends and the future value of their shares at the same rate r (so that they are indifferent between investing in public firms' shares and lending capital to the public firms). Thus the total value of the public firm's shareholders can be computed as

$$\Pi = \max_{k,l}\{-wl - k + \frac{(1-x)(F(k,l) + \Pi)}{1+r}\} \tag{4}$$

[3]Chari et. al. (2005) determine the firm's price at IPO in a similar way.

The nominator of the second term in (4) is the expected total payoff to the to the shareholders in the next period: with probability $(1-x)$ the public firms remain operational, and then the shareholders receive dividends $F(k,l)$ and can resell their shares at a market value Π.

By the first order conditions to (4), optimal capital and labor inputs in public firms k^* and l^* are found from

$$F_k(k^*,l^*) = \frac{1+r}{1-x} \quad \text{and} \quad F_l(k^*,l^*) = \frac{1+r}{1-x}w \tag{5}$$

Plugging k^* and l^* back into (4), we can express the value of the public firm as

$$\Pi = \frac{(1-x)F(k^*,l^*) - (1+r)(k^*+wl^*)}{r+x}. \tag{6}$$

Such determination of Π has important implications for entrepreneurial decision of whether or not to go public when the opportunity arrives. An owner of the firm faces two different frictions: borrowing constraints and incomplete insurance. By remaining in the possession of his business forever, an entrepreneur would get the flow of payoffs, the present expected value of which (discounted at the interest rate r at which the business owner can make risk-free savings) is equal to Π at most. If the entrepreneurs goes public, he receives Π upfront without any risk. Thus such pricing scheme predicts that any business owner would go public once such opportunity becomes available, i.e. $I_{IPO}(a) = 1$ for all $a \geq 0$.[4]

On one hand, such prediction of the model seems to be somewhat extreme and could be relaxed by introducing some costs associated with IPO.[5] On the other hand, some empirical studies have documented that firms indeed go public whenever they can (e.g. Zingales ???) and there is consensus in the existing IPO literature that IPOs are primarily driven by two motives: access to credit and risk diversification (ADD REFERENCES). As discussed above, the same two motives are present in our model. In addition, this stylized prediction of our model simplifies analysis and

[4]Chari et al. 2006 obtain similar prediction in their benchmark model without capital gain taxes.

[5]It is hard to make analytical predictions regarding IPO decision since the shape of the the value functions is endogenous. Our numerical results seem to indicate that introducing a fixed IPO cost leads to counterfactual prediction that only relatively new firms would go public.

allows us to describe in an easy way the relationship between flows of public and private equity.

2.4 Competitive Equilibrium

Each agent's current state can be described by a bundle $s = (a, y) \in \mathbf{S} = \mathbb{R}_+ \times \{w, e\}$, where y is the agent's occupation in the current period. The optimal policies to (1)-(3) together with the probabilities η, x and p define a law of motion over the agent's states $Q : \mathbf{S} \times \mathcal{S} \to [0, 1]$, $\mathcal{S}$ is the σ-algebra of the subsets of $\mathbf{S}$: $Q(s, S) = Prob\{s'(s) \in S\}$. The exact expression for $Q(s, S)$ is straightforward to write down, but it is quite messy, that is why we omit it for expositional purposes. Denote by λ a stationary probability measure on $(\mathbf{S}, \mathcal{S})$ associated with $Q(s, S)$.[6] Then the fraction of entrepreneurs in the economy can be measured by

$$E = \int_{\{(a,y)\in\mathbb{R}_+\times\{w,e\}|y=e\}} \lambda(da \times dy), \tag{7}$$

their total capital investment (corresponding to the total amount of private equity in the economy) is

$$K = \int_{\{(a,y)\in\mathbb{R}_+\times\{w,e\}|y=e\}} k(a) \quad \lambda(da \times dy), \tag{8}$$

the entrepreneurs' joint investment in public equity is

$$A_e^d = \int_{\{(a,y)\in\mathbb{R}_+\times\{w,e\}|y=e\}} b(a) \quad \lambda(da \times dy), \tag{9}$$

and the aggregate amount of labor they hire is

$$L^d = \int_{\{(a,y)\in\mathbb{R}_+\times\{w,e\}|y=e\}} l(a) \quad \lambda(da \times dy). \tag{10}$$

Similarly, workers' total investment in public equity can be measured as

$$A_w^d = \int_{\{(a,y)\in\mathbb{R}_+\times\{w,e\}|y=w\}} \frac{a'(a)}{1+r} \quad \lambda(da \times dy). \tag{11}$$

[6] We do not make any claim regarding existence or uniqueness of λ.

In order to determine the total amount of public equity available for consumers' investment, we first need to find the total number of public firms. This step is simplified by the fact that all entrepreneurs wants to go public when the opportunity arrives. Thus in the steady state the number of public firms can be computed as

$$N = \frac{p}{x}E, \tag{12}$$

where E is given by (7). Each of those firms issues Π ownership shares and rents k^* units of capital. That is why in the equilibrium the amount of public investment should be equal to

$$A^s = \frac{p}{x}E(\Pi + k^*). \tag{13}$$

Now we are ready to formally define a competitive equilibrium of this model.

DEFINITION 1 *A recursive competitive equilibrium consists of the value functions* $V_w(a)$*,* $V_e(a)$ *and* $V_o(a)$*, the policy functions* $a'(a)$*,* $b(a)$*,* $k(a)$*,* $l(a)$*,* k^* *and* l^**, and prices* w*,* r *and* Π *such that*

1. *given prices* (w, r, Π)*, value functions and policy functions solve the decision problems (1)-(3);*

2. *given prices* (w, r)*,* k^* *and* l^* *solve maximization problem (4) and the resulting* Π *is given by (6);*

3. *labor and public equity markets clear, i.e.* $L^d + N * l^* = 1 - E$ *and* $A^d_e + A^d_w = A^s$*.*

3 Some properties of the equilibrium allocation

Obviously, the equilibrium price of a risk-free asset cannot exceed the time discount rate, $\beta(1 + r) \leq 1$ (if the opposite were true, there would be no endogenous upper bound on agents' wealth and the demand for public equity would be infinite). Moreover, due to the presence of uninsured risk, we should expect that the strict inequality would hold in a stationary equilibrium allocation, $\beta(1 + r) < 1$.

Figure 1 illustrates a typical example of value and policy functions obtained in a stationary equilibrium of our model. Several things should be pointed out:

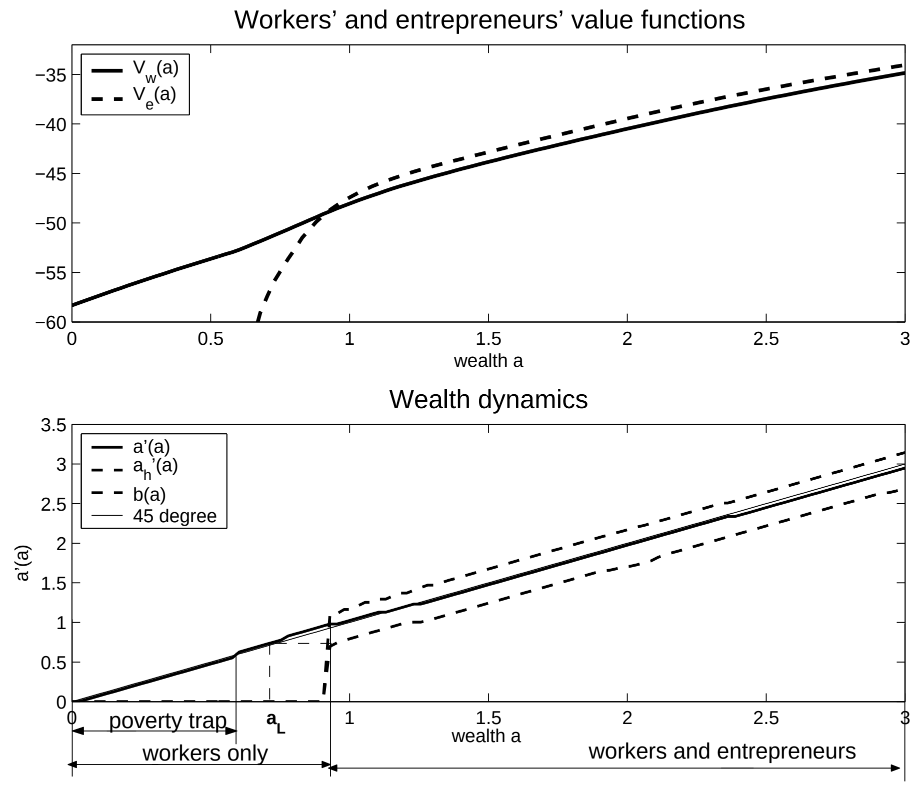

Figure 1: Properties of the solution to (1)-(3).

1. **Occupational choice:** As in any standard occupational choice model, relatively poor agents become workers and the rich ones become entrepreneurs whenever they have an opportunity. This happens because poor agents cannot invest sufficient amount of capital in their business to make it productive enough to compensate for giving up a positive wage w.

2. **Poverty traps:** Since $\beta(1+r) < 1$, the only incentives for poor workers to choose an increasing wealth profile is an opportunity to open up a business if they come up with an idea. However, if the worker is currently very poor, it would take him too much time to accumulate sufficient amount of wealth so that $V_e(a) > V_w(a)$. Therefore, the potential benefit from becoming an entrepreneur is heavily discounted, and it cannot compensate for the necessary consumption sacrifice which should be made in the current period if this poor worker switches to an increasing wealth profile. Thus the poverty trap might

arise, within which all the agents remain workers forever.[7] They choose a decreasing wealth profile and in the long run end up consuming their wage in every period and make no savings. Obviously, this group of agents does not contribute to the aggregate demand for public equity.

Clearly, the poverty traps do not necessarily appear: if the entrepreneurial opportunity is very profitable, the rate of arrival of ideas η is sufficiently high and the equilibrium interest rate is not too low, even the poorest workers would choose to accumulate wealth in order to eventually open up a business. However, if the poverty trap arises, there are many stationary distributions associated with the transition function $Q(s, S)$: all the agents whose initial wealth falls into a poverty trap region remain there forever. Thus the multiple equilibria can potentially arise in our model. However, since the workers in the poverty trap do not generate any demand for public equity, their fraction does not affect much the relative equilibrium quantities and returns of public and private equity since all the action on the capital markets is coming from the behavior of the agents outside of the poverty trap region.

3. **Wealth dynamics outside of the poverty trap:** The bottom plot of Figure 1 illustrates the wealth dynamics for the workers (solid line) and the entrepreneurs managing their own business (dashed lines). In accordance with our discussion above, the poorest workers outside of the poverty trap choose an increasing wealth profile; since $\beta(1 + r) < 1$, the wealth level of the relatively rich workers declines over time – these workers would can afford becoming an entrepreneur at any moment, so there is no need to make extra savings.

   Entrepreneurs face idiosyncratic risk, thus the evolution of their assets depends on whether their business survives or not. Relatively poor entrepreneurs get richer if their firms remains in business and poorer if it exits. We have "truncated" the plots on Figure 1 at relatively low wealth level in order to clearly illustrate wealth dynamics of poor agents, but the very rich entrepreneurs choose such portfolio that their wealth declines even if their business survives.

   The last thing to notice is that the equilibrium prices would be such that the

[7] Various implications of the poverty traps have been previously discussed in Buera (2006) and Vereshchagina and Hopenhayn (2006).

poorest entrepreneurs end up with the wealth level outside of the poverty trap even if their business closes down. If the opposite were true, all the agents would end up in the poverty trap in the long run, and the aggregate demand for public equity would be equal to zero. Thus the risk-free savings $b(a)$ of the poorest entrepreneur determine the lower bound on the wealth distribution of people outside of the poverty trap (labelled by a_L on Figure 1).

4 Numerical Results

In this section we describe a numerical exercise the goal of which is to analyze the effects of the change in likelihood of going public p on the relative quantities and returns of public and public and private equity. This is not a calibration exercise, at this stage our goal is to choose 'reasonable' parameter values and understand the mechanics of the model.

We need to specify the following parameters: the time discount factor β and the instantaneous utility function $u(c)$, the production technology $F(k,l)$ and the probabilities η (of coming up with an idea), x (of business survival) and p (of being able to go public). If the poverty traps arise, we also need the decide what fraction of population is allocated there.

We set $\beta = 0.96$ (thinking of a period as a year), choose CRRA utility function $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ and set $\sigma = 2$. We choose the decreasing returns to scale production function for $F(k,l) = A(k^{\alpha}l^{1-\alpha})^{\theta}$. If the firm is operated at the optimal size, the expected profit share is equal to $1-\theta$. We set $\theta = 0.92$ as in Bohacek (2006). The capital share is set to $\alpha = 0.34$. The average exit rate in manufacturing is around 10% a year (e.g. Davis and Haltiwanger 1992), so we choose $x = 0.1$.

We can set p in order to approximate the ratio of the value of private equity to the value of public equity reported in Moskowitz and Vissing-Jorgensen (2002). According to their estimates, in 1992 this ratio was around 0.76 (Table 3 on p. 752). In our model, this ratio does not exceed $\frac{Ek^*}{A^s}$, where A^s is defined in (13) (since some of the entrepreneurs are borrowing constrained and own-business investment involves idiosyncratic risk). Using (5), (4) and (12)-(13), we obtain that

$$\frac{Ek^*}{A^s} = \frac{x}{p} \cdot (\frac{1+r}{r+x} \cdot \frac{1-\theta}{\alpha\theta} + 1)^{-1} > 0.76.$$

If we choose the other parameters of the model in such a way that the interest rate is about 3.2% a year, the above inequality implies that $p < 0.0439$. We choose $p = 0.03$.

Finally, η and A are fixed in such a way that the ratio of entrepreneurs as a fraction of population is close to 13% (and thus the total labor supply is $L^s = 0.86$) and the equilibrium interest rate is 3.2%. We simulate the model for the grid of different consider η and A and, focusing on equilibria in which all the agents are outside of the poverty trap, try to match $E = 0.14$ and $r = 0.032$. The following observation helps us to choose the grid values. In a stationary equilibrium the following relationship should hold:

$$xE + (1-x)pE = \eta W,$$

where W is the total number of workers whose wealth a is such that $V_e(a) > V_w(a)$. Obviously, $W < 1 - E$ (since some workers are still saving to become entrepreneurs and prefer to remain workers even if they come up with a business idea) and thus the desired η should exceed $\frac{xE+(1-x)pE}{1-E} = 0.0185$. The best combination of η and A we could find so far is $\eta = 0.033$ and $A = 8.5$. The main properties of the equilibrium benchmark economy are summarized in Table 1.

For the chosen parameter values, the model overpredicts the targeted fraction of entrepreneurs (19% instead of 13%), slightly underpredicts the equilibrium interest rate and the ratio of private to public equity (perhaps, we should have chosen a slightly bigger p). The returns to private equity for each household are computed as the ratio of capital gains to the amount of invested capital. The sixth row of Table 1 reports the private equity returns computed in this way and averaged across all active entrepreneurs. As we can see, the obtained return is extremely high (around 34%).

Moskowitz and Vissing-Jorgensen (2002) also account for foregone earnings (subtract the wage that would have been earned if the agent were working for hire from

Table 1: The effects of an increase in p on equilibrium quantities and prices

	p=0.03	p=0.08
Fract. of entr., E	0.19	0.13
Aver. wealth of entr.	100%	121%
Aver. wealth of workers	86%	110%
Priv. to pub. equity, $\frac{K}{A^s}$	0.79	0.32
Ret. to pub. equity, r	3.1%	3.8%
Av. ret. to priv. equity	34.9%	44.3%
Av. ret. to priv. equity (net of foregone wages)	5.3%	10.8%

the agents' profit) and report that the average return to entrepreneurial activity is approximately the same as the return to public equity. The last row of Table 1 reports the profit recomputed in this way. It is a much more reasonable number, but is still quite high. Of course, it would be impossible to generate approximately equal returns to private and public equity in an occupational choice model because opening up a business in this model is beneficial only if entrepreneurial activity offers a premium. However, some assumptions could be adjusted in order to lower the size of this premium. One possible way would be to assume that entrepreneurs differ in their skills and only the best firms get sold at IPO. One possible way of doing this is to assume that the exit probability x could be high or low, its value is realized at the moment of business start-up and is very persistent. Then, since IPO opportunities arrive infrequently, the public firms would endogenously have on average higher survival rates than privately owned firms, which would drive up their relative return.[8]

The second column in Table 1 reports how the stationary equilibrium allocation adjusts if p increases. According to Moskowitz and Vissing-Jorgensen (2002), from

[8] A related selection mechanism has been used in Campanale (2005) as an attempt to explain the private equity puzzle documented by Moskowitz and Vissing-Jorgnsen (2002). His paper takes the return to public equity exogenous and does not model the transition of private to public firms through IPOs.

1992 to 1998 the ratio of the values of private to public equity has decreased from 0.76 to 0.39. Our model predicts that if IPO opportunities arrive more frequently (p rises from 0.03 to 0.08) this ratio drops from 0.79 to 0.32. Naturally, this decline in the relative quantity of private equity is accompanied by the decrease in the number of entrepreneurs (because the rate of arrival of business ideas η has not changed).

After p had gone up, the total quantity of publicly traded equity had risen by more than 35%. However, this increase has not lead to any significant change in the return to public equity (it went up from 3.1% to 3.8%). This is because the change in p happens simultaneously with the shift in aggregate demand for public equity, which is driven by two major forces. First, as the number of entrepreneurs falls, the number of workers rises, and this naturally drives up the demand for public equity (since public equity is the only workers' investment opportunity). Second, an increase in p produces a significant shift in wealth distribution because the amount of agents who can sell their business through IPO and increase their wealth considerably (by receiving Π upfront) almost doubles. Rows 2 and 3 in Table 1 report that both workers and entrepreneurs became on average 20% richer, and thus their savings in public equity rises. As a result, the return to public equity increases only slightly, from 3.1% to 3.8%. In fact, the relative increase in the return to public equity is much higher – because many more entrepreneurs obtain very large capital gains by going public.

In short, this simple numerical exercise illustrates how the dynamics of relative quantities of and returns to public and private equity, puzzling at first sight, can potentially be explained by changes in occupational structure of population and the distribution of wealth. Of course, some realistic features could be added to our model to make the results more tractable, a careful calibration exercise should be implemented and further empirical prediction of our model should be analyzed. We address these issues in the next (final) section.

5 Final Remarks

Even though the current version of the paper conveys the main intuition of our argument, more work should be dome along the following directions:

1. *To be understood within the current set up:*

   – To make the paper's motivation stronger, we need to complement Moskowitz and Vissing-Jorgensen (2002) evidence by comparing the riskiness of public and private investment for the whole time period (1989-1998). It would also be useful to do a simple numerical exercise calculating what increase in relative volatility of the public equity might explain our motivating facts within a representative agent portfolio choice model.

   – Chari et. al. (2006) find a closed form solution of a very related model in the partial equilibrium model under the assumption of linear utility function. It would be nice to check whether the closed form solution could be found in our general equilibrium environment.

   – In the numerical exercise presented in a paper we completely neglect the presence of the poverty traps. Intuition suggests that since the agents in the poverty trap do not make any savings in the long run, the relative properties of the public and private equity should not be affected by the fraction of people that is 'caught' there. The only effect would occur through the labor market. In short, if we add some people to the poverty trap in the current numerical exercise, the labor supply will rise, the wages will drop and both public and private equity will pay higher returns. Still, it is useful to check the exact quantitative implications.

   – It might be useful to simulate the transition process given that there is readily available data on the dynamics of the public equity and the IPOs. Technically, it should not be difficult to do this given that there is no aggregate uncertainty in the model. However, I am not sure how tractable the results would be given the presence of the dot com boom in the data.

2. *Additional features for more rigorous numerical work:* If we introduce heterogeneity in firms' exit rates (as it was discussed in the previous section), the

model would probably generate closer to the data relative returns to public and private equity. The values of the exit rates could be calibrated to match the data on public and private firms' survival rates.

3. *Open question:* Since Π is determined as the present value of the expected life-time profit flows, there is no choice made by an entrepreneur of whether or not to go public: all entrepreneurs sell their firms whenever the opportunity arrives. Of course, this prediction is too extreme, and I am currently thinking of a more coherent way of modelling IPO decision within the occupational choice framework.

   Two papers have been mentioned in the Introduction that have made some steps in this direction:

   In Clementi (2002) the IPO decision is essentially driven by the exogenous productivity shock (going public involves some fixed cost; thus only the firms, whose expected price at IPO is high enough, go public; the IPO price is determined by the firms' productivity; the productivity level follows an exogenously given submartingale; that's why older entrepreneurs are likely to go public).

   In Bhattacharya and Ravikumar (2001) entrepreneurs go public because they are not allowed to invest in any other asset but their own firm, which operates a decreasing returns to scale technology. Thus a firm gets sold through IPO when its owner becomes so rich that the marginal returns to his investment in own company gets smaller than the return to available outside investment opportunities (exogenous in their model). If within this environment, like in our model, entrepreneurs can hold a portfolio of assets, the main motive for going public disappears.

   Thus it would be interesting to analyze what drives the timing of IPO for an entrepreneur running a firm of a given quality. The IPO literature typically lists four main reasons for going public: relaxing borrowing constraints, risk diversification, established reputation and coming up with significant productivity improvement. The latter is essentially modelled in Clementi (2002), even though he takes such improvements as exogenous. The interaction of the former three factors, to my knowledge, has not been analyzed in a unified framework, though it might create some interesting action. Relaxing the borrowing constraints and diversifying risk is most important for relatively poor agents, whose firms are relatively young and small (because their value function is steeper and they are more risk averse). Establishing reputation (e.g., learning about the firms' quality) requires some time. Thus it might be interesting to analyze the tradeoff between going public early and postponing it for later years within the same model. Analysis like this might produce a number of interesting implications about the composition of public and private firms across different industries and its relation with the industry average IPO timing.

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