Preferences for Risk in Dynamic Models with Discrete Adjustments[*]

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June 2009

Abstract

This paper characterizes the solution to a consumption/savings decision problem in the presence of discrete adjustments. A number of recent studies have suggested that the presence of discrete adjustments may help explain known anomalies of consumer's risk behavior, such as simultaneous purchase of insurance and lotteries or the equity premium puzzle, because they (i) create incentives for gambling and (ii) amplify risk aversion with respect to small risks. This paper argues that even though these predictions naturally arise in static models, they do not necessarily extend to a dynamic setup. We show that (i) the possibility of choosing *when* to make the discrete adjustment can eliminate the gambling motive and that (ii) the agents, who plan to make the discrete adjustment in the future, become more tolerant to small risks than the agents with the same wealth levels in the model without discreteness.

JEL classification: D14, D91, E21, G12

Keywords: Consumption commitments; Adjustment costs; Dynamic discrete choice; Housing; Risk aversion; Equity premium puzzle

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[*] The paper has ben previously circulated under the title "Preferences for Risk in a Dynamic Model with Consumption Commitments".

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1 Introduction

Discrete choice is a common feature of many economic decision problems. Examples are numerous, including occupational choice,[1] marriage and fertility decisions, technology adoption, consumption choice in the presence of fixed adjustment costs or borrowing constraints,[2] and many others.[3] Thus it is not surprising that various implications of discrete decisions have been broadly discussed in economic literature. In particular, it has been noted that the discrete nature of certain adjustments may help explain some anomalies of consumers' risk behavior documented in the data, namely the willingness to take lotteries, as well as high risk aversion with respect to moderate income (or wealth) risks.

A typical argument goes as follows. If income is one of the determinants of the decision variable that is adjusted discretely, then the consumer's indirect utility function is not globally concave in income, implying that in some ranges income lotteries may become beneficial. At the same time, small income changes typically do not induce the consumer to make a discrete adjustment and are fully absorbed by other, more flexible, variables (such as consumption of non-durable goods). Thus the discreteness of some choice variables amplifies the variability of the others in response to moderate income shocks. If the consumer is risk averse with respect to these flexible variables (as in the case with consumption of non-durable goods), discrete adjustments increase the welfare costs of moderate income risk, magnifying the degree of risk aversion with respect to moderate income shocks.[4]

A number of authors have argued that such effects of discrete choice on consumers' risk attitudes may help understand some of the puzzling observations in different areas of economics. For example, the discreteness of occupational choice may help explain the rise in consumer bankruptcies over the last decades (Livshitz et. al. 2007) and excess risk taking by entrepreneurs (Vereshchagina and Hopenhayn, 2009). Alternatively, the existence of bounded from zero transaction fees associated with adjustments of durable goods – which endogenously generates discrete adjustments of durables – may contribute to explaining the equity premium puzzle (dates back to Grossman and Laroque 1990), and justify why many employment contracts feature rigid wages while incorporating the possibility of unemployment (Postlewaite et al. 2006).

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[1] For example, the choice between work for hire versus entrepreneurship, or employment versus retirement.

[2] For example, the choice of durable consumption goods, such as housing.

[3] Observe that while some adjustments are discrete by their nature, such as occupation or fertility decisions, the discreteness of others may be generated endogenously due to the presence of fixed adjustment costs.

[4] A formalization of these arguments in different environments can be found, for example, in Chetty and Szeidl (2007).

Given the numerous applications, it is important to understand to what extent the above mentioned effects of discrete choice on consumers' risk attitudes are robust to various changes in economic environments. This paper argues that even though these predictions naturally arise in static models, they do not necessarily hold in a dynamic setup. The reason is that in a dynamic environment consumers can decide not only which one of the discrete options to choose right now, but also whether to switch to a different option in the future. This may give rise to *transitory behavior*: some consumers choose to gradually accumulate (or reduce) their asset holdings and make the discrete adjustment only when their wealth reaches a certain level. Such transitory behavior may arise if the consumer is borrowing constrained and needs to accumulate certain amount of wealth in order to implement the discrete adjustment,[5] if the available options are discrete by nature and the consumer switches between the two levels over time in order to average the life-time consumption of the discrete good,[6] or if any changes in economic environment occurred after the initial decision has been made and a fixed transaction cost must be paid to implement the adjustment.[7]

The role of such transitory behavior is the main focus of this paper. We show that it affects consumers' attitudes towards risk in two ways. First, the possibility of choosing different discrete options during lifetime helps consumers to smooth out the non-concavities in the indirect utility function and, therefore, may reduce, and even completely eliminate, the gambling motive. Second, in dynamic environments the consumers can respond to moderate income shocks by adjusting their asset holdings in addition to the flexible variables with respect to which they are risk averse (such as consumption of non-durables). For the consumers who are saving (or dissaving) in order to implement the discrete adjustment in the future, doing this may be particularly attractive. This is because for such consumers small variations in wealth translate into small variations in the timing of the discrete adjustment, and, if the life-time utility is time-separable, these variations do not necessarily generate welfare losses. In fact, we show that, while for those consumers, who do not plan to implement the discrete adjustment in the future, the degree of risk aversion with respect to small income shocks is magnified by the presence of discrete adjustments (as in the static models), the *consumers who exhibit transitory behavior become more risk tolerant*.

The implication of our findings is that discrete choice may be less helpful in under-

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[5] For example, a downpayment must be paid to purchase a house or a set-up cost must be incurred to start own firm or adopt a new technology.

[6] For example, the consumer can average his life-time work hours by choosing when to retire if part-time employment is not possible.

[7] For example, a persistent increase in income may induce the consumer to start saving in order to move to a bigger house.

standing the anomalies of consumer's risk behavior than it has been suggested by previous theoretical studies. Take, for example, the idea that the presence of transaction fees for housing may help explain the equity premium puzzle. Our results suggest that the overall effect of inflexibility of housing adjustments on consumers' risk preferences is ambiguous: it reduces the welfare cost of moderate risks for some consumers (those who adopt transitory behavior) and magnifies the risk aversion of the others (those who do not plan to switch to a different house in the future). Hence, the effect of housing adjustments costs on 'aggregate risk attitudes' (e.g. aggregate demand for risky assets) depends on how likely the transitions are to occur. In the last section of the paper we develop a series of numerical exercises and argue that the transitory behavior is likely to arise and could have significant impact on the economy's aggregates. Our observations are consistent with the recent quantitative results by Stokey (2009)[8], who finds that the presence of transaction costs for housing adjustments is not likely to be a significant component in explaining the equity premium puzzle.[9] In some sense, the analysis in our paper provides a theoretical rationale for Stokey's numerical results.

Another interesting application of our findings pertains to the effect of borrowing constraints on risk attitudes. Recall that in an environment without discrete adjustments, borrowing constraints magnify the cost of uninsured income risk because consumption of constrained agents is more sensitive to negative income shocks than consumption of unconstrained agents. In the models with discrete choice, borrowing constraints may cause transitory behavior by requiring that a certain amount of wealth is accumulated before the discrete adjustment is implemented (e.g. the downpayment for a house, or cost needed to start own firm or pay the technology adoption fee). We show that, depending on the relationship between the time discount and the interest rates, the consumers exhibiting such transitory behavior may be risk lovers, risk neutral or risk averse. Had there been no borrowing constraints, such transitory behavior would not arise since the consumer would be able to implement the desired discrete adjustment immediately, and the consumer would be

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[8] Stokey (2009) also uses the methodology developed in Grossman and Laroque (1990) to provide an analytical characterization of the housing choice in continuous time models under the assumptions of CES utility function, Brownian investment returns and no borrowing constraints.

[9] Several other studies, such as Fratantoni (2001) and Fukushima (2005), investigate numerically whether inflexible housing adjustments can contribute to explaining the equity premium puzzle. They use very different frictions to generate infrequent housing adjustments. Instead of introducing fixed adjustment costs, they alow housing changes to occur either in certain time periods or with some probability. In other words, the possibility of choosing the moment of housing adjustment, is not taken into account in these studies, which, in accordance with our findings, overestimates the role that inflexible housing adjustments play in explaining the equity premium puzzle.

risk averse independently of the values of the time discount and the interest rates. Thus, in the presence of discrete adjustments, borrowing constraints make some consumers *more* risk tolerant. This is an interesting observation, which, to our knowledge, has not been made in the previous literature.

Our analysis also uncovers a novel relationship between the patience of the consumers exhibiting transitory behavior and their risk attitudes. We show that the relationship between the time discount rate and the risk-free interest rate is crucial for shaping the risk preferences of such consumers. When the two rates are equal to each other, these consumers are risk neutral with respect to small income (or wealth) shocks. When these rates are different, the consumers are either risk averse or risk lovers, depending on whether they are saving or dissaving prior to implementing the discrete adjustment. In particular, the less patient is the consumer who is saving in order to switch to a new option in the future, or the smaller is the risk-free interest rate paid on his savings – the more risk tolerant he turns out to be. While the relationship between the risk-free interest rate and risk tolerance is very intuitive,[10] the relationship between patience and risk aversion is less plausible. The formal reason is that that, for the consumers exhibiting transiting behavior, small income (or wealth) shocks lead to variations in the timing of a discrete adjustment. The discrete adjustment generates a discrete increase in the instantaneous utility function. Time-separability and exponential discounting in the lifetime utility creates convexity in consumer's preferences with respect to the timing of the discrete adjustment (provided that time discount factor is below 1). [11] The smaller is the time discount rate, the bigger is the degree of this convexity, and the more risk-tolerant is the consumer. Thus, among the consumers who save in order to implement the discrete adjustment in the future, the less patient ones also turn out to be more risk-tolerant.

Our paper builds on a recent work by Chetty and Szeidl (2007), who analyze the effects of transaction costs associated with housing adjustments on consumers' risk preferences. They consider an environment where, after the initial housing choice is made, a permanent income shock is realized, and housing adjustments may become desirable. Within this model, the authors analyze how the presence of housing adjustment costs affects the consumers' risk attitudes with respect to further unanticipated income shocks. Even though Chetty and

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[10] Investing in a safe asset is an outside option to taking risk (investing in a risky asset); thus higher risk-free rate reduces the willingness to opt out for a risky option.

[11] Note, however, that such convexity with respect to the timing of the adjustment does not imply that the consumer must be risk lover with respect to income (or wealth) shocks. This is because randomizing over income (or wealth) may, depending on the size of the the risk-free interest rate, increase or reduce the expected time needed to accumulate the amount of wealth at which the adjustment becomes optimal.

Szeidl's model is dynamic, it is built in such a way that all the housing adjustments (if any) are made in the first period only, and the transitive behavior – which is the main focus of our paper – does not arise. In particular, Chetty and Szeidl (2007) assume that the risk-free interest rate is equal to zero and that consumers can borrow against their future income. Under these assumptions, the consumers have no incentives to delay housing adjustments.[12] Relaxing either one, however, would create a motive for transitory behavior. Namely, if the interest rate is positive, late switches reduce the present value of the fixed adjustment cost; if borrowing constraints are present, the consumers might need some time to accumulate sufficient funds to finance the switch. Since these mechanisms are shut down in Chetty and Szeidl (2007), all housing adjustments in their model happen only in period zero, the transitory behavior never occurs and their results regarding the effects of housing adjustment costs on consumers' risk attitudes are similar to the predictions obtained in static models. We extend Chetty and Szeidl's environment by allowing for a positive interest rate and/or borrowing constraints, which enables us to analyze the effects of transitory behavior.

Following Chetty and Szeidl (2007), our results are derived in a dynamic perfect foresight model. On the one hand, it may sound unusual that we discuss the implications for consumers' risk attitudes in the model in which there is no uncertainty. On the other hand, such approach is quite instructive because by analyzing the shape of the indirect utility function in the deterministic environment we are able to understand whether the consumers would be willing to undertake risk (i.e. invest in risky assets) if such option was available. Alternatively, from the solution of this deterministic problem we can immediately derive the welfare losses associated with unexpected permanent income (or wealth) shocks. Thus when we say that discrete adjustments make the consumer less (more) risk averse, we mean that the welfare losses associated with unexpected permanent socks are smaller (higher) in the environment with discrete adjustments than without.

Finally, our paper makes a methodological contribution. To describe consumers' behavior in the presence of discrete adjustments, we need to solve a dynamic model of discrete choice. It is well known that, in general, models of this type are hard to characterize analytically. In particular, it is difficult to establish the single crossing property of the value functions associated with the available discrete options.[13] One sufficient condition that would guar-

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[12] This is true if the time discount rate and the interest rate are equal to each other. Such assumption is made in Chetty and Szeidl (2007) and it considerably simplifies the analysis. Since in their paper the interest rate is set to zero, the time discount rate is also zero, which implies that Chetty and Szeidl's model had to be formulated in finite horizon.

[13] Establishing single crossing property allows to derive the cutoff rules for the state variable which characterize the agents' discrete choice.

antee such single crossing (and which is often used in dynamic discrete choice models[14]) is the monotonicity of the difference between (i) the value of switching to the new option in the current period and (ii) the value of switching to the same option in the following period. Unfortunately, this condition does not hold in many discrete choice models, where the instantaneous payoff is bounded from above – as it is typical in the models of consumer choice. We show that in this class of models standard recursive methods can be used to derive a much weaker condition guaranteeing single crossing. It turns out that instead of establishing the monotonicity of the difference between the two value functions (switching now and switching in the following period), it is enough to verify that these value function cross only once, which can be easily done in our setup. Moreover, this sufficient condition can be easily verified not only in our model with housing adjustment costs but also in other similar environments with perfect foresight, such as occupational choice[15] or costly technology adoption[16] models. Hence, even though our results are derived in a housing model, they also can be immediately applied to other economic problems. For instance, they can generate a number of testable predictions about the behavior of the consumers who make savings to start their own business, upgrade to a new technology, go to school, retire, etc.

The rest of the paper is organized as follows. Section 2 describes the modeling environment and presents our main theoretical results. Section 3 develops a series of numerical exercises to analyze whether transiting behavior might have quantitatively important consequences in our framework. Section 4 discusses our results and outlines the directions for further research.

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[14] See, for example, Dixit and Pindyck (1993)

[15] Occupational choice models with uncertainty are heavily used in macroeconomic models. The examples include Lucas (1974), Quadrini (2000), Bohacek (2006), Cagetti and DeNardi (2006) and others. The examples of occupational choice models with perfect foresight are studied in Buera (2006), Vereshchagina and Hopenhayn (2006).

[16] See, for instance, Greenwood and Jovanovic (1990)and Khan and Ravikumar (2002).

2 Theoretical Analysis

2.1 The Model

Consider an agent who receives utility from two consumption goods, food and housing. The agent's life-time utility is given by:

$$\sum_{t=0}^{+\infty} \beta^t u(c_t, h_t),$$

where $\beta \in (0,1)$ is the time discount factor, and the instantaneous utility in period t is derived from the flow consumption of food c_t and housing h_t. Assume that $u(\cdot,\cdot)$ is defined on $\mathbb{R}^2_+$, is bounded from above, is strictly increasing, strictly concave, and satisfies Inada conditions.

Suppose that in every period the agent receives income $y \geq 0$, which can be spent on consumption goods or stored in a risk-free asset which offers interest rate r. In most of the analysis below we assume that $\beta(1+r) = 1$, and will later discuss the implications of relaxing this assumption. The consumer's total asset holdings a_t might be subject to the borrowing constraint $a_t \geq \underline{a}$ (if $\underline{a} = -\infty$ there are no borrowing limits and the transversality condition $\lim_{t\to+\infty} \frac{a_t}{(1+r)^t} = 0$ is imposed instead). Denote the agent' initial asset holdings by a_0.

Food consumption c_t is flexible and can be adjusted at no cost. In contrast, a fixed adjustment cost $\eta > 0$ must be paid if the agent switches from one housing level to another. The presence of the transaction cost η guarantees that the housing adjustments, if they occur, are discrete. At this stage, we assume that the agent's initial housing consumption h_0 is exogenously given.[17] In the numerical exercise in the last Section we endogenize it by introducing permanent income shocks at $t = 0$ and allowing consumers to choose h_0 optimally prior to the realization of uncertainty in y. In this Section we analyze the consumer's problem after the initial housing decision has already been made and all uncertainty has been resolved, i.e. h_0 is a state variable alongside with a_0.

Notice that, since $\beta(1 + r) = 1$ and there is no further uncertainty, the consumer will make at most one housing adjustment over the course of lifetime (after which he will maintain constant consumption levels of food and housing for the rest of his life). This considerably simplifies the characterization of the consumer's optimal choice because we can easily describe its properties after the housing adjustment is made. Thus the decision problem of the

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[17] Since h_0 is fixed, the adjustment cost η may be interpreted as a function of h_0, i.e. a fraction of the total housing value.

consumer with initial housing commitment h_0, initial wealth a_0 and per period income y can be written as the choice of the food consumption and wealth profiles $\{c_t\}_{t=0}^{+\infty}$ and $\{a_t\}_{t=1}^{+\infty}$, the moment of housing adjustment T as well as the level of housing consumption h^* to which the consumer switches in period N:

$$
\begin{aligned}
V(a_0, h_0) = \max_{\{c_t, a_t\}, h^*, N \in \{0,1,2,\ldots,+\infty\}} \sum_{t=0}^{N-1} \beta^t u(c_t, h_0) &+ \sum_{t=N}^{+\infty} \beta^t u(c_t, h^*) \\
\text{s.t. } c_t + h_0 + \frac{a_{t+1}}{1+r} &\le a_t + y, \quad 0 \le t < N, \\
c_t + h^* + \eta + \frac{a_{t+1}}{1+r} &\le a_t + y, \quad t = N, \\
c_t + h^* + \frac{a_{t+1}}{1+r} &\le a_t + y, \quad t > N, \\
a_t &\ge \underline{a}, \quad t \ge 0, \\
a_0 \text{ and } h_0 &\text{ are given.}
\end{aligned}
\tag{1}
$$

Note that the agent might decide to switch to a new house right away by setting $N = 0$ (in which case the first set of the budget constraints is irrelevant) or to remain in his initial house h_0 forever by setting $N = +\infty$.

Decision problem (1) is an extension of the decision problem studied in Chetty and Szeidl (2007). The environment in this paper differs from Chetty and Szeidl (2007) in two respects: we let the interest rate r be positive and also allow for the possibility of borrowing constraints. Either of these features might induce the consumer to wait for a while prior to adjusting housing consumption, either in order to decrease the present value of the adjustment cost η (if $r > 0$) or in order to accumulate enough funds to finance the switch (which might be relevant when the borrowing constraints are binding). In contrast, Chetty and Szeidl (2007) analyze the model with $r = 0$ and without the borrowing constraints.[18] That is why in their framework the consumers make housing adjustments (if any) only in period 0.

2.1.1 Restricted decision problem: no housing adjustments in $t \geq 1$ and no borrowing constraints

As a benchmark, it is useful to briefly discuss what the solution to the consumer's decision problem would be if delaying housing adjustments was not allowed and there were no borrowing limits. In this case, the consumer chooses between staying in his initial house forever

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[18] Similar to this paper, Chetty and Szeidl (2007) also assume that $\beta(1 + r) = 1$ in order to eliminate the incentives for repeated housing adjustments. Since they set $r = 0$, it must be that $\beta = 1$ and that is why they study a finite horizon framework.

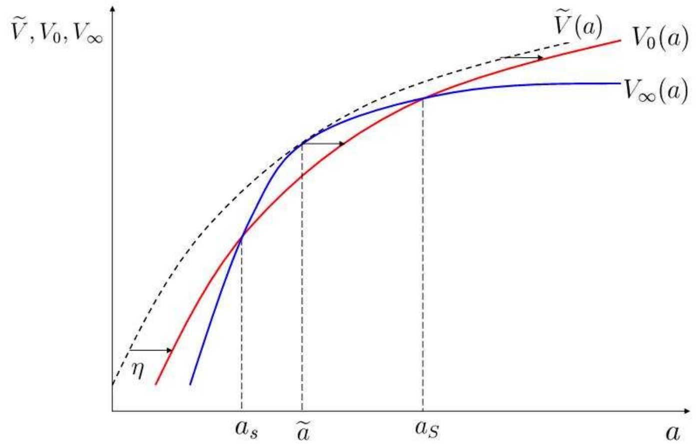

Figure 1: An (s,S) property of the consumer's housing choice when housing adjustments are made only at $t = 0$.

and switching to a new house of optimal size h^* immediately. Denote the value of staying in the initial house h_0 by $V_\infty(a)$. It can be expressed recursively as

$$V_\infty(a) = \max_{a'}\{u(a + y - h_0 - \frac{a'}{1+r}, h_0) + \beta V_\infty(a')\}. \tag{2}$$

If the consumer could adjust housing consumption costlessly, his value $\widetilde{V}(a)$ would be

$$\widetilde{V}(a) = \max_{h^*}\{V^*(a; h^*)\}, \tag{3}$$

where $V^*(a; h^*)$ is the value of staying in house h^* forever:

$$V^*(a; h^*) = \max_{a'}\{u(a + y - h^* - \frac{a'}{1+r}, h^*) + \beta V^*(a'; h^*)\}. \tag{4}$$

Obviously, $V_\infty(a) = V^*(a; h_0)$.

Since the consumer has to pay cost η for housing adjustments, the value $V_0(a)$ of switching to a new house immediately can be found as

$$V_0(a) = \max_{a', h^*}\{u(a + y - h^* - \eta - \frac{a'}{1+r}, h^*) + \beta V^*(a'; h^*)\}. \tag{5}$$

Note also that $V_0(a) = \widetilde{V}(a - \eta)$.

The consumer's value is then given by $\max\{V_\infty(a), V_0(a)\}$. The value functions $\widetilde{V}(a)$, $V_\infty(a)$ and $V_0(a)$ are illustrated on Figure 1. Observe that $\widetilde{V}(a) \geq V_\infty(a)$ for all a, and the values $\widetilde{V}(a)$ and $V_\infty(a)$ are tangent at the wealth level $\widetilde{a}$ at which housing consumption h_0 would be chosen had there been no adjustment costs. Since $V_0(a)$ is obtained by shifting $\widetilde{V}(a)$ to the right by η, $V_0(\widetilde{a}) < V_\infty(\widetilde{a})$, implying that the consumer would choose not to make housing adjustments in the neighborhood of $\widetilde{a}$. Later on (in Proposition XX) we will verify that $V_0(a)$ intersects $V_\infty(a)$ twice, at $a_s < \widetilde{a}$ and $a_S > \widetilde{a}$, and that $\max\{V_\infty(a), V_0(a)\}$ is not smooth at these intersection points. Thus the consumer's housing decision has an (s, S) property: he stays in the initial house if $a \in [a_s, a_S]$, moves into a smaller house if $a < a_s$ and moves into a bigger house if $a > a_S$.

The shape of the resulting value $\max\{V_\infty(a), V_0(a)\}$ of the agent describes his risk preferences because it determines the welfare costs of unexpected permanent wealth (or income, since there are no borrowing limits) shocks. Since this value function has kinks at a_s and a_S, the agent has a gambling motive in some wealth ranges. At the same time, inside the (a_s, a_S) interval, the agent's value function appears to have higher curvature[19] than $\widetilde{V}(a)$. This is obviously true at a_0, and Chetty and Szeidl (2007) show that, under some assumptions on the instantaneous utility function $u(c, h)$[20], this also holds for any other wealth level inside (a_s, a_S). The implication is that the presence of housing adjustment costs magnifies the degree of risk aversion in the (s, S) band.

In what follows we show that if the agent is allowed to delay housing adjustments, his value function changes in two ways. First, it becomes globally concave and the gambling motives disappear. Second, the interval, within which the agent's risk aversion is magnified relative to the environment without adjustment costs, shrinks, and there appear the intervals where the housing adjustment costs make the consumer more risk tolerant. In the last Section of the paper, we illustrate in a series of quantitative exercises that such intervals are likely to be large relative to the intervals where the risk aversion is magnified, and that their presence may have a significant impact on the risk aversion at the 'aggregate level'.

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[19] A measure of curvature of a value function $v(a)$ that is used in Chetty and Szeidl (2007) to evaluate risk preferences is the the coefficient of relative risk aversion (CRRA) $\gamma(a) = v''(a)a/v'(a)$.

[20] Namely, Chetty and Szeidl (2007) require that $u(c, h)$ is either homogeneous of some degree or is separable in two goods and has constant CRRA in food consumption.

2.2 Solution of the full dynamic problem: recursive approach

The solution approach described in this Section can be applied to a broader class of dynamic discrete choice models with perfect foresight, such as occupational choice or technology adoption. Thus the analysis below is somewhat more general than is needed to solve our housing model, which might make it attractive to a larger set of readers, especially those working with similar dynamic models with discrete choice.

It is convenient to explicitly reformulate the consumer's decision problem (1) as the choice of the moment of switching to a new house. For brevity, we drop h_0 and denote by $V(a)$ the value of the consumer who has not moved to a new house yet. It can be represented as

$$V(a) = \max\{V_\infty(a), V_0(a), V_1(a), V_2(a), V_3(a), ...\}, \tag{6}$$

where $V_t(a)$ is the value of the consumer who plans to move into a new house in t periods. Given $V_0(a)$, the sequence of the value functions $\{V_t(a)\}_{t=1}^{+\infty}$ can be determined recursively:

$$V_{t+1}(a) = \max_{a' \geq \underline{a}}\{u(a + y - h_0 - \frac{a'}{1+r}, h_0) + \beta V_t(a')\} = \mathbb{T}V_t(a), \quad t \geq 0. \tag{7}$$

Equation (7) says that the consumer, who plans to move into a new house in $t+1$ periods, chooses his savings optimally and in the next period continues with the value of the consumer who plans to move into a new house in t periods. Obviously, the value $V_\infty(a)$ of the consumer who remains in house h_0 forever (defined in (8)) is the fixed point of the operator $\mathbb{T}$:

$$V_\infty(a) = \lim_{t \to +\infty} \mathbb{T}^t V_0(a). \tag{8}$$

Such recursive representation helps to fully characterize the solution to the consumer's decision problem (1) and describe the properties of the value function $V(a)$. The crucial step in characterizing $V(a)$ is establishing the following Lemma:

Lemma 1 *Let $\mathcal{C}([\underline{a}, +\infty))$ be a set of bounded, strictly increasing, strictly concave and continuously differentiable functions on $[\underline{a}, +\infty)$, and let $\mathbb{T} : \mathcal{C}([\underline{a}, +\infty)) \to \mathcal{C}([\underline{a}, +\infty))$ be an operator defined in (7). Suppose that $F \in \mathcal{C}([\underline{a}, +\infty))$ and $G \in \mathcal{C}([\underline{a}, +\infty))$ have at most one intersection and $F(a) > G(a)$ for all $a > a^* \geq \underline{a}$ (see Figure 2). Then*

(i) $\mathbb{T}F(a)$ and $\mathbb{T}G(a)$ cannot have more than one intersection and $\mathbb{T}F(a) > \mathbb{T}G(a)$ for sufficiently large a; let $\widehat{a}^* = \min\{a \geq \underline{a} : \mathbb{T}F(a) \geq \mathbb{T}G(a)\}$;

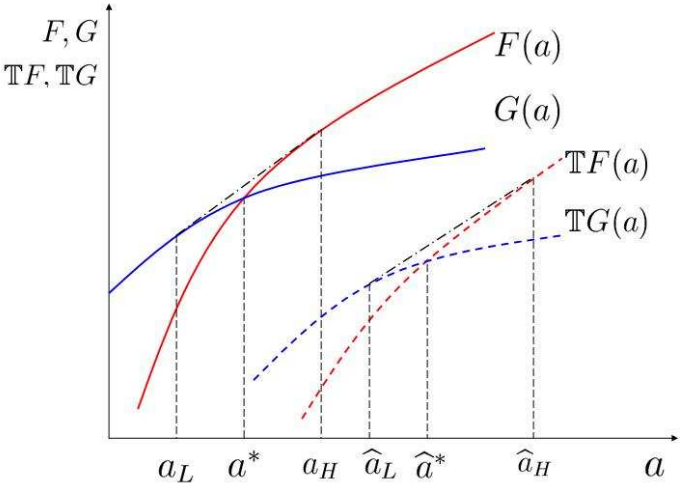

Figure 2: Illustration to Lemma 1

(ii) $a'_F(a) \geq a^*$ *for all* $a \geq \widehat{a}^*$ *and* $a'_G(a) \leq a^*$ *for all* $a < \widehat{a}^*$*, where* $a'_F(a)$ *and* $a'_G(a)$ *are the optimal saving policy rules of the agents maximizing* $\mathbb{T}F(a)$*and* $\mathbb{T}G(a)$ *respectively;*

(iii) denote by a_L *and* $a_H \geq a_L$ *the wealth levels at which* $F(a)$ *and* $G(a)$ *are tangent with their common tangent line, and by* $\widehat{a}_L$ *and* $\widehat{a}_H$ *the corresponding wealth levels for the pair of functions* $\mathbb{T}F(a)$ *and* $\mathbb{T}G(a)$*. If* $a_L > \underline{a}$ *and* $\widehat{a}_L > \underline{a}$ *then* $a'_G(\widehat{a}_L) = a_L$ *and* $a'_F(\widehat{a}_H) = a_H$*.*

These results hold for any value of $\beta(1+r) > 0$ *(not only* $\beta(1+r) = 1$*).*

The results of Lemma 1 are very intuitive. The first statement says that if the choice between the two options $F(a)$ and $G(a)$ is described by a unique threshold rule then the decision problem of the agent who chooses whether to continue with $F(a)$ or $G(a)$ in the future is also described by the unique threshold rule for the agent's current wealth level. The second statement says that the savings policy of the agent is consistent with his future choice between $F(a)$ and $G(a)$: for instance, if the agent prefers $\mathbb{T}F(a)$ in the current period then he would save so much that in the next period he would indeed prefer to choose option $F(\cdot)$. The last statement follows directly from envelope and first order conditions and implies that the agent's optimal saving policy maps the tangent points of the value functions (corresponding to different options) with their common tangent line into the points with similar property. The proof of Lemma 1 is in the Appendix.

Note that Lemma 1 does not require that the difference between $F(a)$ and $G(a)$ is monotone in a. In contrast, such assumption is commonly used in discrete choice literature to establish single crossing of $\mathbb{T}F(a)$ and $\mathbb{T}G(a)$ (e.g. Dixit and Pindyck 1993 or Greenwood and Jovanovic 1990). It is indeed straightforward to verify that, given the structure of the operator $\mathbb{T}$, if $F'(a) \geq G'(a)$ were satisfied for all a then $\mathbb{T}F'(a) \geq \mathbb{T}G'(a)$ would also hold for all a (implying single crossing).[21] However, if $F(a)$ and $G(a)$ correspond to the agent's value functions, they naturally inherit the properties of the instantaneous utility function, and, if the latter is bounded from above, it would follow that $\lim_{a\to+\infty} F(a) - G(a) = 0$. Thus in our model, as well as in other models where instantaneous payoff is bounded from above[22], the differences between the two value functions associated with different options would not be monotone. Lemma 1 shows that single crossing of $\mathbb{T}F(a)$ and $\mathbb{T}G(a)$ can still be established in these environments, as long as $F(a)$ and $G(a)$ have at most one intersection (a weaker condition than monotonicity of their differences). It is also important to point out that the proof of Lemma 1 is quite general, it does not rely on the structure of our particular housing model, and uses standard recursive arguments. Thus we believe that the result of Lemma 1 may be of particular interest to the broader set of readers working with dynamic models of discrete choice.

We can now use Lemma 1 to provide a complete characterization of $V(a)$. The first statement of Lemma 1 implies that if $V_0(a)$ and $V_1(a)$ have at most one intersection then any two consecutive functions $V_t(a)$ and $V_{t+1}(a)$ from the sequence $\{V_t(a)\}_{t=1}^{+\infty}$ defined by (7) would also have at most one intersection. The second statement of the Lemma 1 is used to argue that the corresponding sequence of the cutoff levels must be monotone as long as $V_{t+1}(a) > \max\{V_\infty(a), V_0(a), V_1(a), ..., V_t(a)\}$ for some $a \geq \underline{a}$. Thus we can formulate the following Proposition:

PROPOSITION 1 *Let $\mathcal{C}([\underline{a}, +\infty))$ be a set of bounded, strictly increasing, strictly concave and continuously differentiable functions on $[\underline{a}, +\infty)$, let $\mathbb{T} : \mathcal{C}([\underline{a}, +\infty)) \to \mathcal{C}([\underline{a}, +\infty))$ be an operator defined in (7), and let V_∞ be the fixed point of this operator. Suppose that $V_0 \in \mathcal{C}([\underline{a}, +\infty))$ satisfies the following conditions:*

(a) $V_0(a)$ and $\mathbb{T}V_0(a)$ have a unique intersection at $a_1^* \geq \underline{a}$, i.e. $V_0(a_1^*) = \mathbb{T}V_0(a_1^*)$;

(c) $V_0(a)$ and $V_\infty(a)$ have at most one intersection.

Then $V(a) = \max\{V_\infty(a), V_0(a), V_1(a), V_2(a), V_3(a), ...\}$, where $V_{t+1} = \mathbb{T}V_t$ for all $t \geq 0$, has the following properties:

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[21] This implication follows directly from the envelope and first order conditions.

[22] Such assumption is often made in macroeconomic models for technical reasons and holds for commonly used utility functions, such as constant relative risk aversion $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma > 1$ or exponential $u(c) = 1 - e^{-ac}$ with $a > 0$.

(i) if $V_\infty(a) < V_0(a)$ for sufficiently large a then there exists $a^* \le a_1^*$ such that

$$V(a) = \begin{cases} V_\infty(a), & a \in [\underline{a}, a^*] \\ \max\{V_1(a), V_2(a), ...\}, & a \in (a^*, a_1^*) \\ V_0(a), & a \in [a_1^*, +\infty) \end{cases}$$

Moreover, if $a_1^* > a^*$ then there exists a strictly decreasing (possibly infinite) sequence of the cutoff levels $\{a_t^*\}_{t=2}^T \in (a^*, a_1^*)$ such that

$$V(a) = V_t(a) \quad \textit{and} \quad a'(a) \in (a_t^*, a_{t-1}^*] \quad \textit{for all} \quad a \in (a_{t+1}^*, a_t^*], \quad 1 \le t \le T-1.$$

where $a'(a)$ is the optimal savings policy and $a_0^* > a_1^*$.

(ii) if $V_\infty(a) > V_0(a)$ for sufficiently large a then there exists $a^* \ge a_1^*$ (possibly, $a^* = +\infty$) such that

$$V(a) = \begin{cases} V_0(a), & a \in [\underline{a}, a_1^*] \\ \max\{V_1(a), V_2(a), ...\}, & a \in (a_1^*, a^*) \\ V_\infty(a), & a \in [a^*, +\infty) \end{cases}$$

Moreover, if $a_1^* < a^*$ then there exists a strictly increasing (possibly infinite) sequence of the cutoff levels $\{a_t^*\}_{t=2}^T \in (a_1^*, a^*)$ such that

$$V(a) = V_t(a) \quad \textit{and} \quad a'(a) \in (a_{t-1}^*, a_t^*] \quad \textit{for all} \quad a \in (a_t^*, a_{t+1}^*], \quad 1 \le t \le T-1.$$

where $a'(a)$ is the optimal savings policy and $a_0^* = \underline{a}$.

These results hold for any value of $\beta(1+r) > 0$ (not only $\beta(1+r) = 1$).

Proposition 1 implies the agent's housing decision is described by a set of simple cutoff rules: he remains in the initial house forever if his initial wealth is sufficiently small (*'inaction'* region $[\underline{a}, a^*]$), switches to a new house right away if he is sufficiently rich (*'immediate adjustment'* interval $[a_1^*, +\infty)$) and delays switching to a new house if his wealth falls into the intermediate interval (a^*, a_1^*) (*'transitory'* interval). Proposition 1 also says that within the transitory interval the consumer's wealth monotonically adjusts over time, and thus the closer the agent's initial wealth is to a^*, the longer he would remain in his initial house.

Remark 1 *If $\beta(1+r) = 1$ then the sequence $\{a_t^*\}_{t=2}^T$ of the cutoff levels is infinite (i.e. $T = +\infty$) with $\lim_{t\to+\infty} a_t^* = a^*$, provided that either (i) of Proposition 1 holds in combination with $\underline{a} \le a^* < a_1^*$ or (ii) of Proposition 1 holds.*

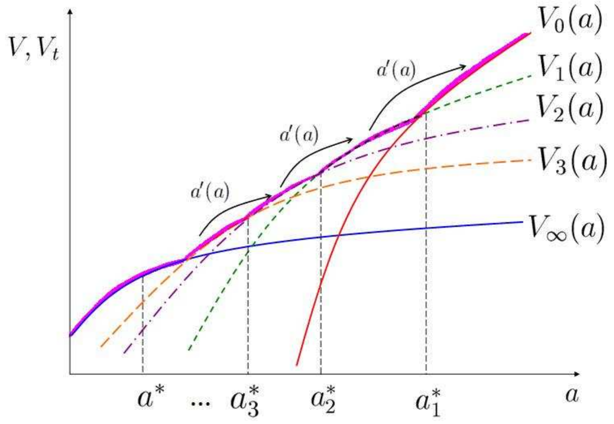

Figure 3: An example of the function $V(a) = \max\{V_\infty(a), V_0(a), V_1(a), V_2(a), ...\}$ and the optimal savings policy $a'(a)$ when $V_\infty(a) < V_0(a)$ for sufficiently large a.

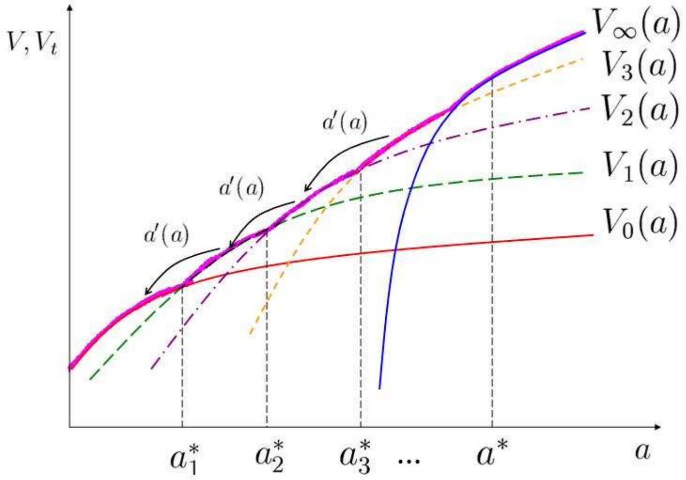

Figure 4: An example of the function $V(a) = \max\{V_\infty(a), V_0(a), V_1(a), V_2(a), ...\}$ and the optimal savings policy $a'(a)$ when $V_\infty(a) > V_0(a)$ for sufficiently large a.

Figures 3 and 4 illustrate examples of the value functions and wealth dynamics in the transitory interval obtained in cases (i) and (ii) respectively. Note that, by the above Remark, since $a^* \neq a_1^*$ and $\underline{a} < a^* < +\infty$, the sequence of the cutoffs $\{a_t^*\}_{t=2}^{+\infty}$ may be infinite, in which case we would have to illustrate infinite sequence of value functions $\{V_t(a)\}_{t=2}^{+\infty}$. In order to keep the Figures cleaner, we have shown only the first three elements of each sequence and indicated that there may be more cutoff levels between a^* and a_3^*.

Finally, it is important to remark that the results of Proposition 1 hold for any value of $\beta(1+r)$ as long as conditions (a) and (b) are satisfied. They also do not rely on the particular specification of the housing model studied in this paper and can be directly applied to other dynamic discrete choice models with perfect foresight. That is why the properties of the agent's discrete choice, as well as the following discussion regarding the effects of transitory behavior of consumers' risk preferences, is not restricted to our housing model, and pertains to many other economic environments.

2.3 The effects of transitory behavior on risk attitudes

This Section provides a more detailed characterization of the agent's value function in the transitory intervals and discusses its implication for risk preferences of the consumers adopting transitory behavior. We start by describing risk attitudes of the agents who are *transiting upwards* – those who are accumulating wealth in order to switch to a new option in the future (i.e. statement (i) or Proposition 1 applies). Then we discuss how the predictions change for the consumers *transiting downwards* – those who are gradually consuming their wealth out, while planning to make a discrete adjustment later on (i.e. statement (ii) or Proposition 1 applies).

2.3.1 Upward transition, $\beta(1+r)=1$

Describing the dynamics of wealth inside the transitory interval (a^*, a_1^*) is crucial for understanding how the possibility of switching to an option $V_0(a)$ in the future affects the curvature of the consumers' value function and, hence, their risk attitudes. Let us look more closely at the relationship between the value functions from the sequence $\{V_t(a)\}_{t=0}^{T}$ when $V_0(a) > V_\infty(a)$ for sufficiently large a, i.e. when statement (i) of Proposition 1 applies.

Denote by $\underline{a}_1$ and $\overline{a}_1 > \underline{a}_1$ the wealth levels at which $V_1(a)$ and $V_0(a)$ are tangent to their common tangent line(see Figure 5).[23] Similarly, denote by $\underline{a}_2$ and $\overline{a}_2 > \underline{a}_2$ the wealth

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[23] Note that $\overline{a}_1 > \underline{a}_1$ is the only interesting case because $\overline{a}_1 = \underline{a}_1$, in conjunction with $\beta(1+r)=1$ would imply that $V_t(\overline{a}_1) = V_0(\overline{a}_1)$ and $V_t'(\overline{a}_1) = V_0'(\overline{a}_1)$ for all $t \geq 1$. Then a limiting argument suggests that

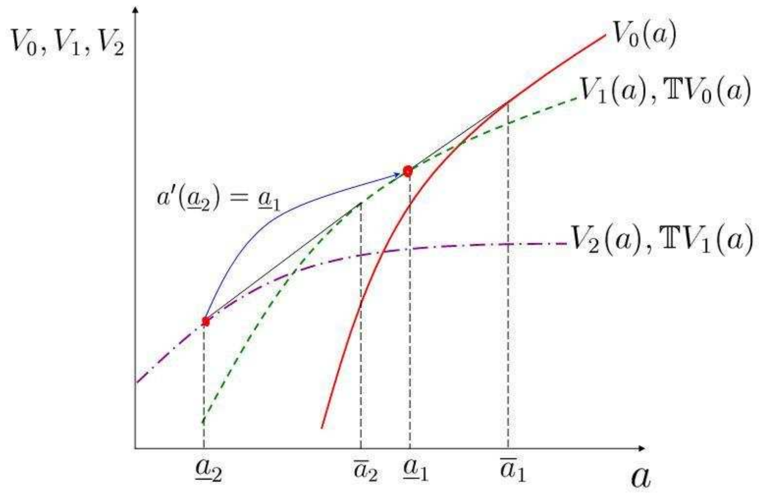

Figure 5: Mutual allocation of $V_0(a)$, $V_1(a)$ and $V_2(a)$ in case (i) of Proposition 1. The relation between $\overline{a}_2$ and $\underline{a}_1$ depends on the value of $\beta(1+r)$.

levels at which $V_2(a)$ and $V_1(a)$ are tangent to their common tangent line. By (iii) of Lemma 1, since $V_2(a) = \mathbb{T}V_1(a)$ and $V_1(a) = \mathbb{T}V_0(a)$, we know that if $\underline{a}_2$ and $\underline{a}_1$ are interior, then $a'(\underline{a}_2) = \underline{a}_1$. Thus, the first order and envelope conditions imply that

$$V_2'(\underline{a}_2) = \beta(1+r)V_1'(\underline{a}_1). \tag{9}$$

At the same time, since $V_2(a)$ and $V_1(a)$ are tangent to their common tangent line at $\underline{a}_2$ and $\overline{a}_2$ respectively, and $\underline{a}_2$ is interior, it must be true that

$$V_2'(\underline{a}_2) = V_1'(\overline{a}_2). \tag{10}$$

When $\beta(1+r) = 1$, the combination of (9) and (10) implies that $\underline{a}_1 = \overline{a}_2$ since $V_1(a)$ is strictly concave, and, therefore, $V_0(a)$, $V_1(a)$ and $V_2(a)$ have a common tangent line. This argument can be applied inductively to all the value functions from the sequence $\{V_t(a)\}_{t=1}^{T}$, which implies that all of them have one common tangent line, and if the agent's initial wealth a_0 coincides with one of the tangent points, then his continuation wealth profile would include all other tangent points to the right of a_0.[24] Moreover, if a^* is interior, (iii) of Lemma 1 can be also applied to a pair of functions $V_\infty(a) = \mathbb{T}V_\infty(a)$ and $V_1(a) = \mathbb{T}V_0(a)$ implying

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$V_\infty(\overline{a}_1) = V_0(\overline{a}_1)$ and $V_\infty'(\overline{a}_1) = V_0'(\overline{a}_1)$, which guarantees that $a^* = a_1^*$ and $V(a) = \max\{V_0(a), V_\infty(a)\}$ is concave.

[24] This argument relies on the assumption that the asset levels $\underline{a}_t$ are interior for all $t = \overline{1,T}$. It is easy to verify that this is indeed true since $\underline{a}_t = \underline{a}$ implies that $V_k(a) < \max\{V_0, .., V_t(a)\}$ for all $k \geq t+1$.

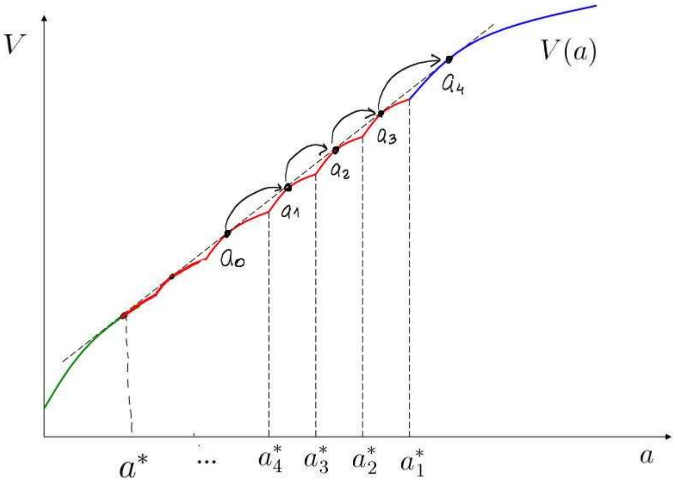

Figure 6: The properties of $V(a)$ and $a'(a)$ if $\beta(1+r)=1$ and (i) of Proposition 1 applies.

that $V_\infty(a)$ is also tangent to the common tangent line of $\{V_t(a)\}_{t=1}^T$. In addition, since in this case $\lim_{t\to+\infty} a_t^* = a^*$, their tangent point occurs at exactly a^*. These observations are summarized in the Proposition below.

PROPOSITION 2 *Suppose that $V_0(a)$ and operator $\mathbb{T}$ satisfy the conditions of Proposition 1. Suppose also that $\beta(1+r)=1$ and (i) of Proposition 1 holds. Then all the value functions from the sequence $\{V_t(a)\}_{t=1}^T$ have a common tangent line, which is also tangent to $V_\infty(a)$ at a^* if $a^* > \underline{a}$.*

Figure 6 illustrates the shape of the agent's value function and the properties of his wealth dynamics inside the transitory interval when $\beta(1+r)=1$. Two observations emerge right away. First, if the consumer's initial wealth a_0 coincides with one of the tangent points inside (a^*, a_1^*), he does not have a gambling motive at any period in his life. Second, $V_0(a)$ appears to be almost linear inside (a^*, a_1^*), implying that the agents with wealth inside this interval is risk neutral with respect to mean-preserving wealth shocks of certain types.[25] These observations are in sharp contrast with the predictions of the restricted model discussed in Section XX, where discrete housing adjustments create gambling motive and magnify the risk aversion of all the consumers remaining in their initial house.

Note, however, that due to time discreteness, small kinks occur at each cutoff level a_t^*, $0 \le t \le T$, implying that (a) if the consumer's initial wealth does not coincide with

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[25] For example, if a_0 corresponds to one of the tangent points then in any of the first N periods of his life the consumer would not mind taking a fair lottery randomizing over any combination of the wealth levels $\{a^*, a_0, a_1, ..., a_N\}$.

any of the tangent points, he would still like to take a small wealth lottery to eliminate the local non-concavity[26], and (b) if the consumer's initial wealth does no coincide with any of the kink points, he would be risk averse with respect to very small wealth risks[27]. However, the arguments presented above do not depend on the length of the time period. Thus we can always shorten the time period, thereby allowing consumers to make wealth adjustments more frequently (doing this would only make the model more realistic).[28] Under such modification, the intervals within which $V(a)$ is strictly concave would shrink. A limiting argument can then be used to show that as the length of the time period converges to zero (i.e. the model becomes a continuous time model), the small kinks disappear and the consumer's value function becomes linear inside the interval (a^*, a_1^*).

To sum up, we have argued that if $\beta(1+r) = 1$ then the possibility of choosing when to implement the discrete adjustment eliminates the incentives to take wealth lotteries and creates transitory wealth intervals, within which the agent's risk aversion is reduced relative to the environment where all the goods can be adjusted flexibly.

2.3.2 Upward transition, $\beta(1+r) \neq 1$

Recall that, even though our housing model is formulated for $\beta(1+r) = 1$, Proposition 1 holds for any values of $\beta(1+r)$. Thus we actually can discuss how risk attitudes of the consumers planning to switch to option $V_0(a)$ (such that $V_0(a) > V_\infty(a)$ for sufficiently large a) would change if $\beta(1+r) \neq 1$. Even though this discussion does not have any bearing in our housing model, it sheds light on an interesting relationship between agents' patience and risk attitudes, which may be relevant in other models of discrete choice.

Figures 8 and 7 illustrate illustrate typical shapes of $V(a)$ for $\beta(1+r) > 1$ and $\beta(1+r) < 1$ respectively. Observe that (9) and (10) hold for any $\beta(1+r)$. Correspondingly, $\beta(1+r) > 1$ would result in $\underline{a}_1 > \overline{a}_2$, while $\beta(1+r) < 1$ would lead to $\underline{a}_1 < \overline{a}_2$. This implies that as the length of the time period shrinks, those consumers who save in order to switch to $V_0(a)$ in the future become risk-averse when $\beta(1+r) > 1$ and risk-lovers when $\beta(1+r) < 1$. Obviously, the gambling motive, which is present in the restricted model from Section XX,

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[26] Note that the welfare benefits of such lottery are likely to be small.

[27] Note, however, that such consumer would not necessarily be more risk averse than the consumer with the same wealth level in the model without housing adjustment costs. The reason is that varying a inside each interval (a_t^*, a_{t+1}^*) is equivalent to varying a at the moment of the discrete adjustment, after η is paid and the consumer flexibly chooses the new housing level.

[28] When we make the time period shorter, we need to adjust the model's parameters correspondingly. For example, if we split each period into n equal sub-periods, we need to set $\widehat{\beta} = \beta^{1/n}$, $\widehat{r} = (1+r)^{1/n} - 1$ and $\widehat{y} = \frac{1+r}{r}\frac{\widehat{r}}{1+\widehat{r}}y$.

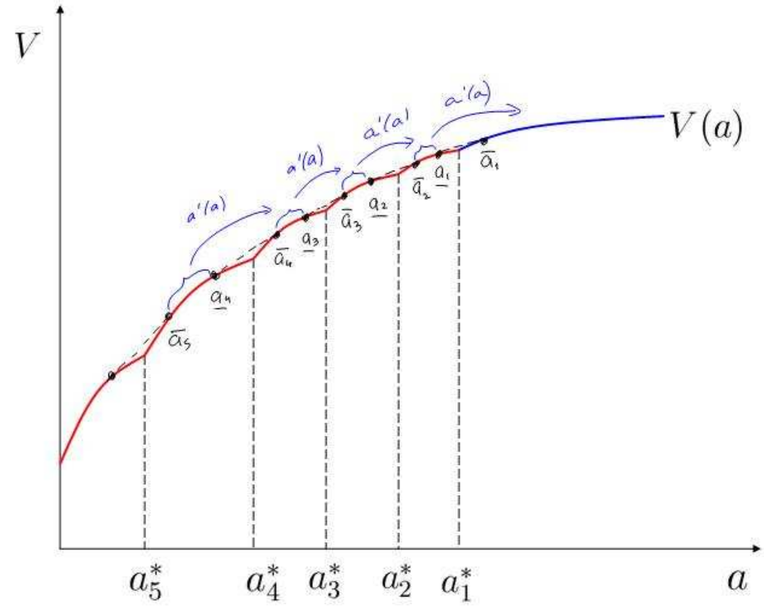

Figure 7: The properties of $V(a)$ and $a'(a)$ if $\beta(1+r) > 1$ and (i) of Proposition 1 applies.

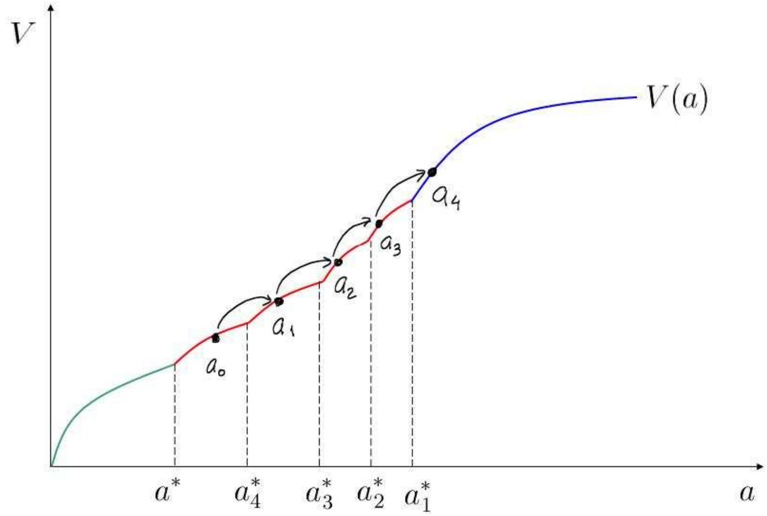

Figure 8: The properties of $V(a)$ and $a'(a)$ if $\beta(1+r) < 1$ and (i) of Proposition 1 applies.

disappears in the former case and persists in the latter. Note also that even though $V(a)$ is concave within (a^*, a_1^*) when $\beta(1+r) > 1$, the consumers in our model might still be less risk averse than in the environment without consumption commitments (in the sense that a lower premium would be required to make them willing to take an unexpected wealth lottery).

2.3.3 The role of $\beta(1+r)$ in determining consumers' risk attitudes

At first glance, it is somewhat surprising that risk attitudes (risk averse/risk loving/risk neutral) of the consumers during the transition to a new option are solely determined by the value of $\beta(1+r)$ and do not depend on the curvature of the instantaneous utility function $u(\cdot, h)$ or of the value function $V_0(a)$. To better understand the intuition behind this finding, we construct two examples suggesting why $\beta(1+r)$ plays such an important role and then explain why the lessons learnt from these examples could be applied to the general version of our model.

Example 1: Wealth accumulation and utility gains

Consider the agent who saves $s > 0$ in period $t = 0$ and by the time when the total cumulative return on his savings becomes equal to Y, the agent can take an action which would raise his life-time utility by Δ (measured in utils). Suppose that in period 0 the agent is allowed to either save s in the risk-free bond or to take a fair lottery and then save its outcome in a risk-free bond. If the lottery is successful, the agent would be able to experience the utility jump Δ sooner; if the outcome of the lottery is not successful, the utility jump would occur later. In order to figure out whether the agent would be willing to take the lottery first, let us compute the present value $PV(s)$ of the utility gain associated with the future utility jump Δ if the agent saves s in a risk-free bond.

For brevity, let us assume (for this part only) that the time is continuous. The agent would accumulate amount Y by time period x such that $s \exp(rx) = Y$. The present value of the utility jump then would be equal to $PV(s) = \exp(-\rho x)\Delta$ (where $\beta = \frac{1}{1+\rho}$), which could be rearranged as

$$PV(s) = \exp(-\rho x)\Delta = (\exp(-rx))^{\rho/r}\Delta = (s/Y)^{\rho/r}\Delta.$$

Notice that $PV(s)$ is strictly concave in s if $\rho < r$ ($\beta(1+r) > 1$), strictly convex if $\rho > r$ ($\beta(1+r) < 1$) and is linear if $\rho = r$ ($\beta(1+r) = 1$). Thus the consumer would prefer to invest in a risk-free bond in the first case, to take the lottery in the second, and would be indifferent between the two options in the third case. Recall that in our benchmark model exactly the same conditions on $\beta(1+r)$ determine whether the agent is risk averse, risk lover or risk neutral while he is saving in order to move to a bigger house.

The predictions in this example are driven by the interaction between two forces. First, exponential discounting implies that the agents are risk lovers with respect to the timing of the utility gain (since its present value $\exp(-\rho x)\Delta$ is convex in x). Second, exponential return on savings implies that the mean-preserving lottery over initial wealth increases the expected waiting time x till the utility gain occurs (since $x = \frac{\ln Y - \ln s}{r}$ is convex in s). While the first force creates incentives for risk taking, the second acts against it. When the consumers are sufficiently impatient, the former dominates because the degree of convexity of $\exp(-\rho x)\Delta$ rises, and the consumers become risk lovers.

Example 2: Separable $u(c,h)$, no borrowing constraints

Now we illustrate that the previous example can be mimicked by a special case of our benchmark model. Suppose that the utility function is separable in consumption and housing, $u(c,h) = v_c(c) + v_h(h)$, and that there are no borrowing constraints. Then the decision problem of the agent can be rewritten as:

$$\begin{aligned} &\max_{\{c_t\},T} \sum_{t=0}^{+\infty} \beta^t v_c(c_t) + \frac{v_h(h_0)}{1-\beta} + \beta^T \frac{v_h(h^*) - v_h(h_0)}{1-\beta} \\ &\text{s.t.} \quad \frac{1+r}{r} y + a_0 = \sum_{t=0}^{+\infty} \frac{c_t}{(1+r)^t} + \frac{1+r}{r} h_0 + + \frac{1}{(1+r)^T} [\eta + \frac{1+r}{r}(h^* - h_0)]. \end{aligned} \tag{11}$$

The similarities between decision problem (11) and Example 1 studied above become obvious once we realize that the consumer's behavior can be interpreted in the following way. The agent borrows against his future income in period 0 and opens two risk-free bank accounts: the savings on the first bank account will be used to finance the stream of consumption expenses $\{c_t\}$ and housing payments of size h_0 throughout his life; while the savings on the second bank account (of the initial size $s = \frac{1}{(1+r)^T}[\eta + \frac{1+r}{r}(h^* - h_0)]$) are used to finance the switch from h_0 to h^* in period T. At the time of the switch the agent's life-time utility would jump up by $\Delta = \frac{v_h(h^*) - v_h(h_0)}{1-\beta}$. Thus, according to our conclusions in Example 1, the agent should strictly prefer to invest his savings in the second bank account in a safe asset if $\beta(1+r) > 1$ and would be risk loving if $\beta(1+r) < 1$.

General intuition

These two examples suggest that the risk attitudes of the consumers inside the intermediate interval (a^*, a_T^*) are explained by the fact that the savings decision of the consumer who eventually plans to switch to a new option could be separated into two parts. More specifically, this agent makes savings for two different purposes: (i) to smooth the marginal utility of his flexible consumption $u_1(c, h_t)$ (which is the same as the marginal value $V'(a)$) over time and (ii) to raise his life-time utility level at the moment when the switch happens. The second type of savings governs the agent's risk preferences during the path of wealth accumulation towards the switch; that is why the findings from Example 1 also apply in a general model.

The above observations uncover an interesting relationship between patience and attitudes towards risk: while saving in order to switch to a more attractive option less patient agents also tend to be more risk tolerant. All the results presented up to now can be applied in a wide variety of dynamic discrete choice models, for which conditions of Proposition 1 can be verified. For instance, if applied to an occupational choice model with borrowing constraints, it suggests that less patient workers should be willing to take more risky jobs (or make more risky investments) if they are planning to become entrepreneurs in the future. Alternatively, our findings can also be used in a costly technology adoption model to argue that, while saving in order to switch to a more productive technology, the firms (or countries) with lower discount factor should be more willing to undertake risky projects. It would be interesting to see whether these predictions find support in the data and, if they do, to study their further implications.

2.3.4 Downward transition

All the steps of the above analysis can be applied in a straightforward way to characterize risk attitudes of the consumers who are *dissaving* while planning to switch to $V_0(a)$ in the future. The results are symmetric: such consumers are risk averse along the transition path if $\beta(1 + r) < 1$, risk neutral if $\beta(1 + r) = 1$ and are risk lovers if $\beta(1 + r) > 1$. Similarly, Proposition **??** can be reformulated for this case: by optimally choosing the moment of switching to a smaller house and the path of wealth during the transition, these consumers are able to smooth out the kink in the value function induced by the discreteness of housing choice if $\beta(1 + r) = 1$. Comparing the predictions for upward and downward transiting consumers, we conclude that risk loving behavior would arise if during the transition the agent's wealth adjusts in the opposite direction from the one induced by the value of $\beta(1+r)$

in the long run.

2.4 Applying the General Approach to the Housing Model

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In order to characterize the solution to (1), we consider separately two subproblems – the one in which the agent is allowed to switch only to a higher house $h^* > h_0$ and the other in which the consumer can move only to a smaller house $h^* < h_0$ – and then describe how the choice of h^* depends on the initial wealth level. Our main methodological contribution and theoretical results are presented in the next Section.

As it was mentioned earlier, it is convenient to separately analyze two cases – moving into a bigger house and moving into a smaller house. Obviously, in the first case, the constraint $h^* \in \Gamma(h_0) = \{h : h \geq h_0\}$ must be imposed in (5), and in the second case it must be replaced with $h^* \in \Gamma(h_0) = \{h : h_0 \geq h > 0\}$.
XXXX

In order to apply the results obtained in the previous Section in our housing model, we remain to verify that $V_0(a)$ and $V_\infty(a)$ defined in (5) and (8) indeed satisfy conditions (a)-(c) of Proposition 1. The standard dynamic programming arguments imply that both value functions inherit the properties of $u(\cdot,\cdot)$, thus (a) of Proposition 1 holds. However, checking whether (b) and (c) hold is less straightforward. The following Proposition summarizes our findings:

PROPOSITION 3 *Suppose that* $\beta(1+r) = 1$, $\eta < \frac{1+r}{r}h_0$ *and* $u(c,h)$ *is supermodular* ($u_{12}(c,h) \geq 0$) *and* $V_0(a)$ *and* $V_\infty(a)$ *are defined in (5) and (8). Then*

(i) if $\Gamma(h_0) = \{h : h \geq h_0\}$, *the conditions (b) and (c) of Proposition 1 are satisfied and* $V_\infty(a) < V_0(a)$ *for sufficiently large* a*;*

(ii) if $\Gamma(h_0) = \{h : h_0 \geq h > 0\}$ *and no borrowing constraints are imposed* ($\underline{a} = -\infty$), *the conditions (b) and (c) of Proposition 1 are satisfied and* $V_\infty(a) > V_0(a)$ *for sufficiently large* a.

Unfortunately, in the presence of borrowing constraints, conditions (b) and (c) of Proposition 1 might be violated if $\Gamma(h_0) = \{h : h_0 \geq h > 0\}$. The intuitive explanation is very straightforward. In the absence of borrowing constraints, when $\eta < \frac{1+r}{r}h_0$, poor consumers would prefer to move into a smaller house because this frees up some life-time wealth for food consumption (i.e. $V_0(a)$ would be above $V_\infty(a)$ for sufficiently low wealth levels). However, when borrowing is restricted and the consumer's current wealth is close to the borrowing limit, moving to a new house (if feasible) cannot be optimal if the adjustment cost is relatively high (close to or exceeds y) because such decision would considerably lower food consumption in the current period (or even make positive consumption levels not feasible). If this happens, $V_0(\underline{a}) < V_\infty(\underline{a})$ and, correspondingly, $V_0(a)$ and $V_\infty(a)$ might cross more than once. Notice that such an issue does not appear when the consumers are allowed to move only into a bigger house ($\Gamma(h_0) = \{h : h_0 \geq h > 0\}$) because $V_0(a) < V_\infty(a)$ for small a anyway. Thus statement (i) of Proposition 3 holds even in the environment with borrowing constraints.

Also notice that Proposition 3 is formulated for $\beta(1 + r) = 1$. As pointed out in the Appendix, we cannot show that in a general case $V_0(a)$ and $TV_0(a)$ have at most one intersection.[29] One could argue that our failure to establish that $V_0(a)$ satisfies the conditions of Proposition 1 when $\beta(1 + r) \neq 1$ makes our discussion in Section 2.3.3 irrelevant. However, we believe that, in spite of being not applicable to a particular housing model presented in this paper, the lessons learnt in that Section are still instructive because they can be applied to other dynamic discrete choice models.

2.5 Optimal Choice of h^*

To complete the characterization of consumer's behavior, we remain to analyze the choice between the options of moving into a bigger house and moving into a smaller house. We do ithis under the assumption that there are no borrowing constraints since Proposition 1 cannot be applied to describe the behavior of the consumer planning to switch to a smaller house when borrowing constraints are binding.

Figure 9 combines all the value functions needed to describe the choice between transiting to $h^* \geq h_0$ and to $h^* \leq h_0$ on one graph.[30] The bold solid line plots $V_\infty(a)$. The bold dashed line illustrates the value of switching to an endogenously chosen bigger house immediately $V_0(a|h^* \geq h_0)$. From Proposition **??** we know that the consumer is indifferent between

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[29] This difficulty arises because the continuation values in the decision problems for $TV_0(a)$ and $V_0(a)$ are different: the consumers maximizing $TV_0(a)$ can choose any h^* in the following period, while the consumers maximizing $V_0(a)$ will have to continue with h^* that is chosen today.

In principle, our model can be modified in several ways guarantee that Proposition 3 holds for any value of $\beta(1+r)$. For instance, it would be sufficient to assume that $u(c) = \frac{c^{1-\sigma_c}}{1-\sigma_c} + \mu \frac{h^{1-\sigma_h}}{1-\sigma_h}$; or that after moving out from initial house h_0 the consumer does not have to pay any adjustment costs any more and can be freely adjusting housing consumption for the rest of his life; or that the set of feasible new houses is restricted to just two exogenously given values, $\overline{h^*} > h_0$ and $\underline{h^*} < h_0$.

[30] Figure 9 is at the same time an illustration to a numerical example described in the next Section of the paper.

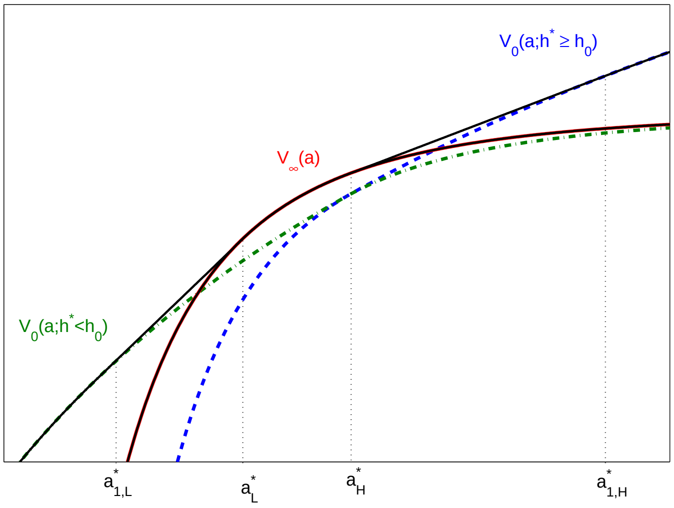

Figure 9: The optimal value V(a), no borrowing constraints

staying in house h_0 forever and transiting to a bigger house at the wealth level a_H^* at which the common tangent line to $V_\infty(a)$ and $V_0(a|h^* \geq h_0)$ touches $V_\infty(a)$. Correspondingly, the bold dash-and-dot line represents the value of switching to a smaller house in the current period $V_0(a|h^* \leq h_0)$. By planning this switch ahead of time, the consumer can eliminate the kink in the value function; he would be indifferent between staying at h_0 forever and start transiting to a new house if his wealth a_L^* is such that the common tangent line to $V_\infty(a)$ and $V_0(a|h^* \leq h_0)$ touches $V_\infty(a)$ at this level.

Notice that the relationship $\min\{V_0(a|h^* \geq h_0), V_0(a|h^* \leq h_0)\} = V_\infty(a-\eta)$ must hold (the former is the value of switching immediately to the house of the same size) and that $\max\{V_0(a|h^* \geq h_0), V_0(a|h^* \leq h_0)\}$ is strictly concave (it is the value of switching immediately to the house of any size). Thus $a_L^* < a_H^*$ and, correspondingly, the intervals of wealth, inside which remaining in the current house forever is dominated by transiting to a bigger house or by transiting to a smaller house, are disjoint.

Thus the consumer's housing choice is described by an (S,s) policy: stay in the initial house if $a \in [a_L^*, a_H^*]$, plan to switch to a smaller house if $a < a_L^*$ and plan to switch to a bigger house if $a > a_H^*$. Among the consumers planning to transit to a smaller house (bigger) house, only the poorest (richest) move right away, the rest stay in house h_0 for a while and switch to a new house later in the future.

2.6 Brief Summary of Theoretical Results

It is useful at this stage to summarize our theoretical findings and emphasize once again why it is important to study consumption commitments in a dynamic environment if one wants to understand their effects on consumers' attitudes towards risk.

In this Section we have shown that in a dynamic model the presence of fixed adjustment costs (giving rise to endogenous consumption commitments) leads to the appearance of transitory wealth intervals, within which the agents choose to consume their initial level of the commitment good in the current period but are planning to adjust it in the future. We found that, even though this future adjustment will be lumpy, it does not generate a kink in the agents' value function; the consumers are able to smooth the kink out by optimally choosing the moment of the adjustment. In contrast, in a static model, the consumer's indirect utility function would always have a kink at the point where the agent is indifferent between consuming his endowment of the commitment good and adjusting it. This lead us to make the first important observation: while in a static environment consumption commitments make some agents willing to take unfair lotteries, the demand for such lotteries might disappear once dynamics is added.

Similarly to the predictions of static models, fixed cost generate an *inaction region* (wealth interval $[a_L^*, a_H^*]$) in our environment. Inside this interval moderate shocks to income or wealth have no effects on the consumption level of the commitment good and are fully absorbed by the changes in the flexible consumption good. Had there been no adjustment costs, the effects of such shocks would be 'spread out' across all consumption goods. Thus, consistently with the results of static models, consumption commitments magnify the welfare cost of moderate risks inside the inaction region.[31] However, we argue that the opposite prediction obtains outside of the inaction region. When $\beta(1+r) = 1$, the value function becomes linear inside the transitory wealth intervals. Had there been no adjustment costs, the value function would be strictly concave.[32] Hence, consumption commitments actually lower the welfare costs of moderate risk for transiting consumers.

Such heterogeneity of risk preferences raises a natural question of how important the 'transitory' effects might be relative to the 'inaction' effects? Or, put differently, should we expect that a significant number of consumers adopt transiting behavior, in which case

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[31] We can easily visualize this argument using Figure 9: if the consumer was able to adjust housing consumption costlessly, his value would be given by $\max\{V_0(a|h^* \geq h_0), V_0(a|h^* \leq h_0)\}$, which has lower curvature inside $[a_L^*, a_H^*]$ than $V_\infty(a)$.

[32] In the absence of adjustment costs the value function would be given by $\max\{V_0(a+\eta|h^* \geq h_0), V_0(a+\eta|h^* \leq h_0)\}$.

it would be essential to account for their presence in order to understand the effects of consumption commitments on aggregate risk attitudes? In the next Section we attempt to address this question using a series of simple numerical exercises based on our model.

3 Numerical Analysis

In this Section we describe a series of simple numerical exercises analyzing whether the transiting behavior is likely to arise in our dynamic model. In general, consumers might be willing to switch to a new house (immediately or with a delay) if their initial housing choice is ‘wrong’. This might happen, for instance, if an income shock arrives in period 0 after the housing commitment is made. From the analysis in the previous Section we should expect that if the income shock is sufficiently large – the consumer will move to a new house right away, if it is quite small – the consumer would not adjust housing consumption at all, and for the intermediate shock values the consumers might choose to remain in the initial house for a while and adjust it in the future.

Intuition suggests that two features of our model might create incentives for such transitory behavior: positive interest rate $r > 0$ and the presence of the borrowing constraints. When $r > 0$, the consumer can decrease the present value of the adjustment cost (and increase the present value of his life-time wealth) by postponing housing adjustment for a while. When borrowing is restricted, it might be too costly (or even not feasible) to pay the adjustment cost η in period 0 and the consumer might choose to wait for a while till he accumulates sufficient resources to finance transition. As it was mentioned earlier, these features are absent from the model in Chetty and Szeidl (2007), therefore transitory behavior never appears in their environment. In what follows we attempt to identify how important each of these factors might be for stimulating transiting behavior.[33]

Parameterization:

We choose the same parameter values as in the benchmark simulation of Chetty and Szeidl (2007) (see column 2 of Table 2). The instantaneous utility function is separable in consumption and housing:

$$u(c,h) = \frac{c^{1-\sigma_c}}{1-\sigma_c} + \mu \frac{h^{1-\sigma_h}}{1-\sigma_h},$$

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[33] There is a short section in Chetty and Szeidl (2007), which argues that borrowing constraints magnify local risk aversion with respect to a particular type of negative income shocks. We complement their discussion by illustrating that borrowing limits can lower risk aversion with respect to positive income shocks.

where $\sigma_c = 4$ and $\sigma_h = 1$. We think of a period as a year and set $\beta = 0.96$ and $r = 0.0417$.[34] Per period income is normalized to $y = 1$ and μ is chosen in such a way that the agent foreseeing constant income for the rest of his life chooses to spend 50% of it on housing. In this case the initial housing commitment is equal to $h_0 = 0.5$ and $\mu = 8$. The adjustment cost η is equal to 10% of the total life-time value of the initial housing commitment and we will perform a comparative statics exercise with respect to this parameter.

Transitions induced by $r > 0$, myopic consumers

To emphasize the role of $r > 0$, we first consider the model without borrowing constraints. The value functions obtained under our parameterizations were shown on Figure 9. Figure 10 emphasizes that there are substantial differences in risk attitudes between the consumers with different wealth level. It plots the percentage risk premium that would be required to make consumers willing to gamble their current period income in a 50/50 lottery in two different models, with and without consumption commitments. If there are no adjustment costs, such risk premium gradually declines with wealth (since $u(c, h)$ has DARA property). When the fixed adjustment cost of size η is imposed, the consumers with the wealth level around zero (where $h_0 = 0.5$ is optimal) become significantly more risk averse than in the model without adjustment costs. At the same time, consistently with our theoretical findings, the consumers in the transitory wealth intervals are risk neutral – they would be willing to take this income lottery even if it does not offer any premium.

Judging from Figures 9 and 10, the transitory intervals seem to be wide in comparison with the the inaction interval. In the benchmark model, an increase in the initial wealth in the range from 1.57 to 8.33 (157% to 833% of annual income) after the initial housing choice is made would induce the consumer's transition towards a bigger house, and a decline in the initial wealth of the size of 182% to 640% of annual income would give impetus to a downward transition. When there are no borrowing constraints, changes in the initial wealth could be replicated by permanent changes in the annual income (since only the total life time wealth is important for consumer's decision). Thus, in our benchmark model, the consumer would start saving for a bigger house if in period zero, after the initial commitment of $h_0 = 0.5$ is made, he receives an unexpected permanent shock to his income in the range from 6% to 34% (i.e. the new income level varies between 1.06 and 1.34). Similarly, a downward transition would start if per period income drops unexpectedly by 7% to 22%.[35]

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[34] Recall that this is different from parameterization in Chetty and Szeidl (2007), where $\beta = 1$ and $r = 0$.

[35] Income shocks of this size are not uncommon in the data (e.g. Kydland 1984); most numerical macroeconomic models with incomplete markets assume that the standard deviation of the annual income is around

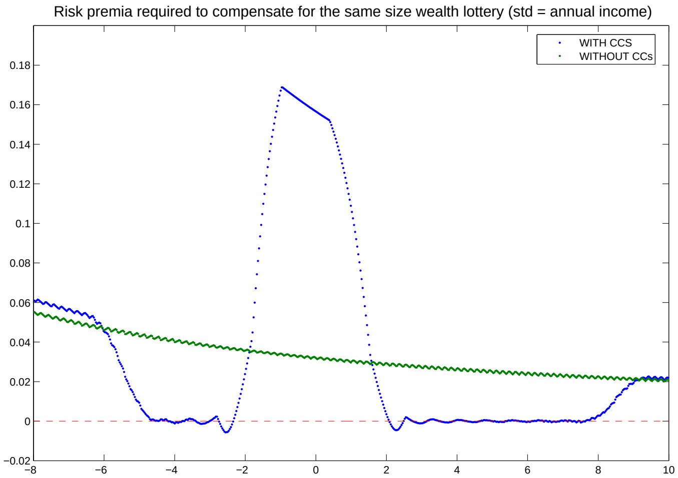

Figure 10: Comparison of the risk premia (Δ) needed to make the consumer willing to gamble their current period income in the models with and without adjustment costs (the lottery is a 50/50 gamble between 0 and $2(y + \Delta)$).

Table 1 presents how the width of the transitory interval adjusts when η falls. As can be seen, even when the adjustment cost is fairly small, the ranges of income shocks which would put the consumer into a transiting interval (and make him locally risk neutral) are significant in comparison with the range of income shocks for which the agent would remains inside the inaction interval (and be more risk averse than in the model without consumption commitments).

Table 1: Income shocks generating transiting behavior ($r > 0$, myopic behavior, no borrowing constraints)

η	% of $\frac{1+r}{r}h_0$	negative y shock	positive y shock
1.250	0.100	(-0.22,-0.07)	(0.06,0.34)
0.625	0.050	(-0.15,-0.05)	(0.05,0.23)
0.312	0.025	(-0.12,-0.03)	(0.04,0.15)

20% (e.d. Hugget 1996).

Table 2: Distribution of *ex post* housing decisions (rational consumers, $r > 0$, no borrowing constraints)

σ	h_0 at $a_0 = 0$	$h^* < h_0$ at $T = 0$	$h^* < h_0$ at $T > 0$	h_0 forever	$h^* > h_0$ at $T > 0$	$h^* > h_0$ at $T = 0$
0.20	0.45	0.09	**0.12**	0.38	**0.30**	0.11
0.13	0.46	0.02	**0.09**	0.54	**0.32**	0.03
0.09	0.47	0.01	**0.04**	0.67	**0.27**	0.01

Table 3: Positive income shocks generating transiting behavior (introducing borrowing constraints)

η	% of $\frac{1+r}{r}h^1$	no constraints	$a_t \geq 0$ is imposed
1.250	0.100	(0.06,0.34)	(0.06,0.81)
0.625	0.050	(0.05,0.23)	(0.05,0.46)
0.312	0.025	(0.04,0.15)	(0.04,0.23)

Transitions induced by $r > 0$, rational consumers

Our first numerical example summarized in Table 1 describes the behavior of the myopic consumers, whose initial housing decision is made under the assumption that there would be no income shocks after h_0 is chosen. If the consumers foresee the possibility of receiving a permanent income shock after the commitment is made, their initial housing choice would be different, and, correspondingly, the transitory intervals would adjust. Thus in our next exercise we model rational consumers, who make their initial housing choice knowing the distribution of income shocks that would arrive afterwards.

Assume that *ex post* annual income is drawn from a normal distribution with mean 1 and variance σ^2. After the shock's initial realization in period 0, the level of income remains constant forever. Table 2 lists the values of initial housing commitments made by rational consumers in period 0 for different levels of σ as well as the distribution across the types of housing adjustments made after the income shock is realized. For instance, when the standard deviation of the income shock is equal to 0.2, the initial housing choice falls to $h_0 = 0.45$ (the consumers insure themselves against the low income realizations). After the income shocks arrive, 9% of the consumers move to a smaller house immediately, 38% remain in their initial house h_0, and 11% decide to move into a bigger house right away. The remaining 42% of the agents start adjusting their wealth in order to eventually move into a new house. Out of them, 12% are transiting downwards and 30% are transiting upwards. As can be seen from Table 2, the smaller is the income uncertainty, the bigger is the fraction of agents who choose to remain in their initial house forever. Even though the fraction of consumers switching right away becomes almost negligible, the amount of transiting consumers remains significant. These observations suggest that the transiting behavior occurring due to $r > 0$ might have a significant quantitative impact on the aggregate demand for risk.

Transitions induced by the borrowing constraints

Finally, we analyze to what extent borrowing constraints can contribute to the appearance of transiting behavior. Given that we were not able to provide the theoretical characterization of downward transitions in the presence of the borrowing constraints (see (ii) of Proposition **??**), we focus in this exercise on upward transitions only. Table 3 compares the ranges of positive income shocks that induce transiting behavior of myopic consumers (the ones who commit to $h_0 = 0.5$ initially) in the models with and without borrowing constraints. As can be seen, imposing the restriction $a_t \geq 0$ considerably broadens the range of permanent income shocks, after the realization of which consumers become risk neutral.

It is also important to point out that this paper describes a situation, in which the presence of the borrowing constraints reduces the welfare cost of risk for a subset of consumers. This is in sharp contrast with the traditional findings in incomplete markets models (e.g. Aiyagari (1993)), where borrowing constraints make agents more risk averse because consumption of constrained consumers must drop if they get hit by a negative shock. In this paper we have shown that borrowing limits, in conjunction with fixed adjustment costs, may generate transitory intervals, within which agents become indifferent to risk (or even like it if $\beta(1 + r) < 1$ and they are transiting upwards).

To sum up, a series of simple quantitative exercises presented in this Section suggests that the occurrence of transitory behavior in the models with fixed adjustment costs (or other features leading to lumpy adjustments) might play an important role in shaping the economy-wide demand for risky assets. At the same time, we must acknowledge that, given that our model is quite stylized, our quantitative results should not be taken seriously. However, we believe that our findings do serve as a motivation for a more advanced numerical project (which we leave for future work).

4 Final Remarks

This paper illustrates that some of the predictions about the effects of consumption commitments on consumers' attitudes towards risk derived from static models do not necessarily hold in a dynamic environment. We showed that (i) consumers can eliminate non-convexities in their value function by delaying housing adjustments and that (ii) the consumers who are planning to switch to a different house in the future may be more risk tolerant than the consumers with the same asset level in the model without fixed adjustment costs. The punch line of our analysis is that consumption commitments (i) do not explain why consumers are often willing to simultaneously buy lotteries and insurance and (ii) have a weaker potential for explaining the equity premium puzzle than it has been suggested in the previous literature.

The major shortcoming of the analysis in the paper is that our theoretical results heavily rely on the fact that the agent's decision problem is deterministic along the transition path towards a new house. Intuition suggests that the qualitative results should be preserved if we add just a bit of uncertainty or introduce very persistent income shocks. Unfortunately, we were not able to extend our methodology to provide analytical characterization of the consumers' decision problem in the presence of idiosyncratic income shocks. However, this problem can undoubtedly be solved numerically. The two major goals of such an exercise would be to (i) quantitatively analyze how the presence of consumption commitments affects the curvature of the consumers' value function and (ii) evaluate whether the fraction of the consumers who become more risk tolerant due to the presence of consumption commitments might be quantitatively significant. If the answer to the latter question is positive, it would be interesting to 'close' this model in a general equilibrium framework and study its asset pricing implications.

Our paper also makes a methodological contribution to the literature studying various dynamic discrete choice models in discrete time. It is well known that, in general, it is quite difficult to establish single crossing of the value functions associated with the available discrete options (which would allow to obtain the cutoff rule). A sufficient condition that would guarantee such single crossing (and which is often used in dynamic discrete choice models, such as Dixit and Pindyck (1993)) is the monotonicity of the difference between (i) the value of switching to the new option in the current period and (ii) the value of switching to the same option in the following period. Unfortunately, this condition does not hold in our model (and it would not hold in many other discrete choice models, where the instantaneous payoff satisfies Inada conditions). However, we can show that, in the presence of a continuous control/state variable (e.g. wealth), a much weaker condition can guarantee single crossing. It turns out that instead of establishing the monotonicity of the difference between the two value functions, it is enough to verify that they cross only once, which can be easily done in our setup. We believe that this result may be useful to other researchers working with dynamic discrete choice models.

5 References

Aiyagari, R. (1993). Uninsured Idiosyncratic Risk and Aggregate Saving. *The Quarterly Journal of Economics,* Vol. 109(3), pp. 659-684

Albuquerque, R. and H. Hopenhayn. Optimal Lending Contracts and Firm Dynamics. *The Review of Economic Studies*, 2004, 71(2), pp. 285-314.

Attanasio, O. (2000). Consumer Durables and Inertial Behaviour: Estimation and Aggregation of (S, s) Rules for Automobile Purchases. *The Review of Economic Studies*, 67(4), pp. 659-677.

Athreya, K. Welfare Implications of the Bankruptcy Reform Act of 1999. *Journal of Monetary Economics*, 49, pp. 1567-1595.

Bailey, M., Olson, M. and P. Wonnacot (1980). The Marginal Utility of Income Does Not Increase: Borrowing, Lending, and Friedman-Savage Gambles. *American Economic Review*, 70, pp. 372-379.

Bohacek, R. (2006). Financial Constraints and Entrepreneurial Investment. *Journal of Monetary Economics*, 53(8), pp. 2195-2212.

Buera, F. (2006). Persistency of Poverty, Financial Frictions and Entrepreneurship. *Unpublished manuscript.*

Cagetti, M. and M. DeNardi (2006). Entrepreneurship, Frictions and Wealth. Journal of Political Economy, 114(5), pp. 835 - 870.

Chetty, R. and Szeidl (2004). Consumption Commitments and Asset Prices. *Unpublished Manuscript.*

Chetty, R. and A. Szeidl. Consumption Commitments and Risk Preferences. *The Quarterly Journal of Economics*, 122, pp. 831–877.

Dixit, A. and R. Pindyck (1993). *Investment Under Uncertainty.* Princeton University Press.

Flavin, M. and S. Nakagawa (2004). A Model of Housing in the Presence of Adjustment Costs: A Structural Interpretation of Habbit Persistence. *NBER Working Paper 10458.*

Friedman, M. and L. Savage (1948). The Utility Analysis of Choices Involving Risk. *The Journal of Political Economy*, 56, pp. 279-304.

Hartley, R. and L. Farrel (2002). Can Expected Utility Theory Explain Gambling? *American Economic Review*, 92, pp. 613-624.

Hugget, M. (1996). Wealth Distribution in Life-Cycle Economies. *Journal of Monetary Economics*, 28, pp. 469-494.

Khan, A. and B. Ravikumar. Costly Technology Adoption and Capital Accumulation. *Review of Economic Dynamics*, 5, pp. 489–502.

Lucas, R. (1978). On the Size Distribution of Business Firms. *Bell Journal of Economics*, 9(2), pp. 508-523.

Marshall, D. and N. Parekh (1999). Can Costs of Consumption Adjustment Explain Asset

Pricing Puzzles? *The Journal of Finance*, 54, pp. 623-654.

Postlewaite, A., Samuelson, L. and D. Silverman (2006). Consumption Commitments and Employment Contracts. *Unpublished manuscript.*

Quadroni, V. (2000). Entrepreneurship, Saving and Social Mobility. *Review of Economic Dynamics*, 3, pp. 1-40.

Rogerson, R. "Indivisible labor, lotteries and equilibrium." *Journal of Monetary Economics,* 21, pp. 3–16.

Shore, S. and T. Sinai (2006). Commitment, Risk and Consumption: Do Birds of a Feather Have Bigger Nests? *NBER Working Paper 11588.*

Vereshchagina, G. and H. Hopenhayn (2006). Risk taking by entrepreneurs. *Unpublished manuscript.*

Warren, E. and A. Tyagi (2003). *The Two-Income Trap.* New York: Basic Books.

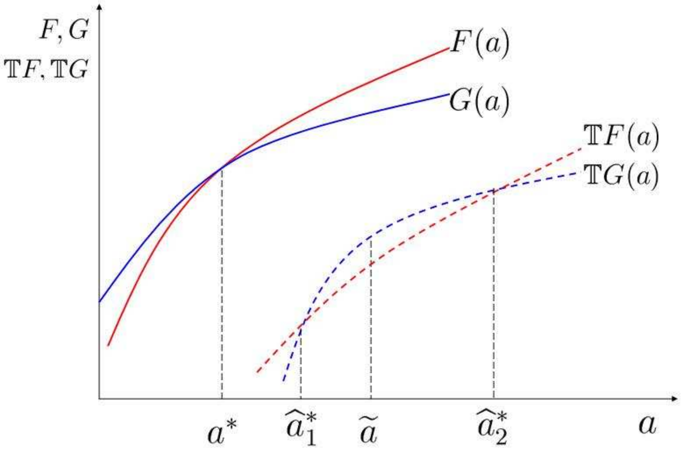

Figure 11: Illustration to the proof of Lemma 1

6 Appendix

Proof of Lemma 1:

It is convenient to establish (ii) of Lemma 1 first. To verify that for any $\widehat{a}$, for which $\mathbb{T}F(\widehat{a}) \geq \mathbb{T}G(\widehat{a})$, it must be true that $a'_F(\widehat{a}) \geq a^*$, we construct the argument by contradiction. Suppose that $a'_F(\widehat{a}) < a^*$. Then, since $F(a) < G(a)$ for all $a < a^*$, it must be that $F(a'_F(\widehat{a})) < G(a'_F(\widehat{a}))$. At the same time, $a'_F(\widehat{a})$ is a feasible saving policy for the consumer maximizing $\mathbb{T}G(\widehat{a})$. Thus

$$\begin{aligned}\mathbb{T}G(\widehat{a}) &\geq u\left(\widehat{a} + y - h_0 - \frac{a'_F(\widehat{a})}{1+r}, h_0\right) + \beta G(a'_F(\widehat{a})) \\ &> u\left(\widehat{a} + y - h_0 - \frac{a'_F(\widehat{a})}{1+r}, h_0\right) + \beta F(a'_F(\widehat{a})) = \mathbb{T}F(\widehat{a}),\end{aligned}$$

which obviously contradicts to $\mathbb{T}F(\widehat{a}) \geq \mathbb{T}G(\widehat{a})$.

A similar argument is used to verify that for any $\widehat{a}$, for which $\mathbb{T}F(\widehat{a}) \leq \mathbb{T}G(\widehat{a})$, the inequality $a'_G(\widehat{a}) \leq a^*$ must hold.

Statement (i) of Lemma 1 is also proven by contradiction. Suppose that $\mathbb{T}F(a)$ and $\mathbb{T}G(a)$ have multiple intersections, labeled by $\widehat{a}_1^*$ and $\widehat{a}_2^*$ on Figure 12, with $\widehat{a}_1^* < \widehat{a}_2^*$. Then there exists $\widetilde{a} \in (\widehat{a}_1^*, \widehat{a}_2^*)$ such that $\mathbb{T}F'(\widetilde{a}) = \mathbb{T}G'(\widetilde{a})$ and $\mathbb{T}F(\widetilde{a}) < \mathbb{T}G(\widetilde{a})$ (it is straightforward to verify that $\mathbb{T}F(\widetilde{a}) > \mathbb{T}G(\widetilde{a})$ for sufficiently lare a). On the one hand, $\mathbb{T}F'(\widetilde{a}) = \mathbb{T}G'(\widetilde{a})$ implies that the agents maximizing $\mathbb{T}F(\widetilde{a})$ and $\mathbb{T}G(\widetilde{a})$ derive the same current period utility. On the other hand, since $\widetilde{a}$ is in between the two intersection points, the properties of the policy functions $a'_F(a)$ and $a'_G(a)$ established above can be used to illustrate that $F(a'_F(\widetilde{a})) > G(a'_G(\widetilde{a}))$,

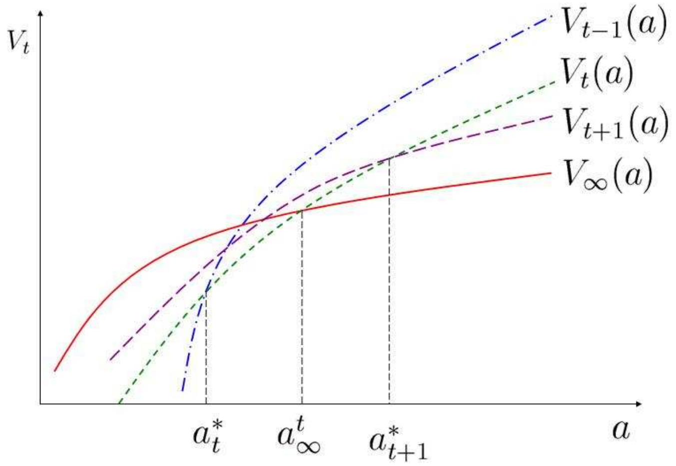

Figure 12: Illustration to Step 1 of the proof of Proposition 1

which leads to an apparent contradiction (namely, it implies that $\mathbb{T}F(\widetilde{a}) < \mathbb{T}G(\widetilde{a})$ cannot hold). To see that $F(a'_F(\widetilde{a})) > G(a'_G(\widetilde{a}))$ indeed holds, observe that $a'_F(a) \geq a^*$ for all $a \geq \widehat{a}_1^*$ (since this is true at $\widehat{a}_1^*$ and, by concavity of $F(a)$, $a'_F(a)$ is monotone) and that $a'_G(a) \leq a^*$ for all $a \leq \widehat{a}_2^*$ (for the same reason). Thus $F(a'_F(\widetilde{a})) > F(a^*) = G(a^*) > G(a'_G(\widetilde{a}))$, which completes the proof. ■

Proof of Proposition 1:

Denote by a_t^* the wealth level at which $V_t(a)$ and $V_{t-1}(a)$ intersect; and by a_∞^t the wealth level at which $V_t(a)$ and $V_\infty(a)$ intersect (conditions (b) and (c) of the Proposition imply that, by (i) of Lemma 1, each pair has at most one intersection). Also denote by $a'_t(a)$ the optimal savings policy of the agent maximizing $V_t(a)$. To prove (i) of Proposition 1, it is sufficient to verify that $a_{t+1}^* < a_t^*$ as long as $V_{t+1}(a) > \max\{V_\infty(a), V_0(a), V_1(a), ..., V_t(a)\}$ for some $a \geq \underline{a}$. This is done in two steps:

Step 1: Verify that if $V_t(a) < \max\{V_\infty(a), V_{t-1}(a)\}$ for all $a \geq \underline{a}$ then $V_{t+1}(a) < \max\{V_\infty(a), V_t(a)\}$ for all $a \geq \underline{a}$.

Suppose that the opposite is true and there exists $t \geq 1$ such that $V_t(a) < \max\{V_\infty(a), V_{t-1}(a)\}$ for all $a \geq \underline{a}$ but $V_{t+1}(a) \geq \max\{V_\infty(a), V_t(a)\}$ for some a. This implies that $a_{t+1}^* \geq a_\infty^t > a_t^*$ (see Figure 12). Correspondingly, $V_{t+1}(a_{t+1}^*) > V_\infty(a_{t+1}^*)$.

First, notice that $a'_{t+1}(a_{t+1}^*) > a_{t+1}^*$ must hold. If the opposite were true then the sequence of the value functions $\{V_t(a)\}_{t=0}^{+\infty}$ would not be converging to $V_\infty(a)$. To see this, notice that $a'_{t+1}(a_{t+1}^*) \leq a_{t+1}^*$ implies that $V_{t+1}(a'_{t+1}(a_{t+1}^*)) \geq V_t(a'_{t+1}(a_{t+1}^*))$ and thus $V_{t+2}(a'_{t+1}(a_{t+1}^*)) \geq$

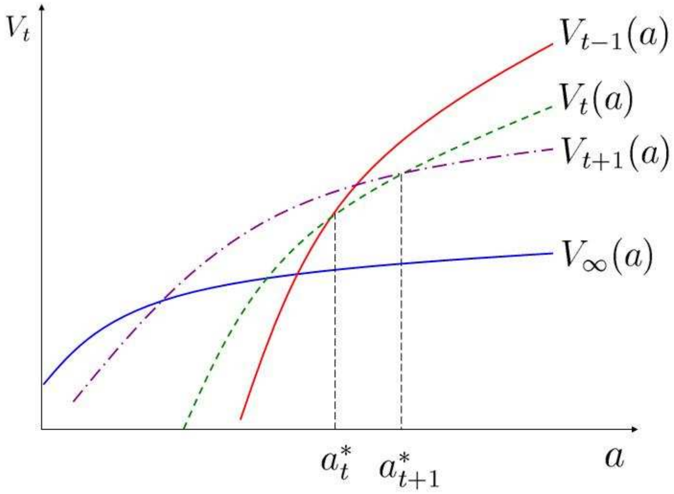

Figure 13: Illustration to Step 2 of the proof of Proposition 1

$V_{t+1}(a'_{t+1}(a^*_{t+1}))$. Since $V_{t+2}(a)$ and $V_{t+1}(a)$ have unique intersection and $V_{t+1}(a) > V_{t+2}(a)$ for sufficiently large a, $V_{t+2}(a) \geq V_{t+1}(a) \geq V_t(a)$ for all $a \leq a^*_{t+1}$. By induction, it follows that $V_{t+k}(a) \geq V_{t+1}(a)$ for all $a \leq a^*_{t+1}$ and $k \geq 2$, which implies that $\lim_{k\to+\infty} V_{t+k}(a^*_{t+1}) > V_\infty(a^*_{t+1})$.

Second, $a'_{t+1}(a^*_{t+1}) \geq a^*_{t+1}$ implies that

$$\begin{aligned} V_{t+1}(a^*_{t+1}) &= u\left(a^*_{t+1} + y - h^* - \frac{a'_{t+1}(a^*_{t+1})}{1+r}, h^*\right) + \beta V_t(a'_{t+1}(a^*_{t+1})) \\ &< u\left(a^*_{t+1} + y - h^* - \frac{a'_{t+1}(a^*_{t+1})}{1+r}, h^*\right) + \beta V_{t-1}(a'_{t+1}(a^*_{t+1})) \leq V_t(a^*_{t+1}), \end{aligned}$$

which contradicts the definition of a^*_{t+1} (namely, that $V_{t+1}(a^*_{t+1}) = V_t(a^*_{t+1})$), implying that $V_{t+1}(a) < \max\{V_\infty(a), V_t(a)\}$ must hold for all $a \geq \underline{a}$.

From Step 1 it follows that, if for some T it happens that $V_{T+1}(a) < \max\{V_\infty(a), V_0(a), V_1(a), ..., V_T(a)\}$ for all $a \geq \underline{a}$, then, for any $k \geq 1$, $V_{T+k}(a) < \max\{V_\infty(a), V_0(a), V_1(a), ..., V_T(a)\}$ for all $a \geq \underline{a}$ and $V(a) = \max\{V_\infty(a), V_0(a), V_1(a), ..., V_T(a)\}$.

Step 2: Verify that $a^*_{t+1} < a^*_t$ for all $t \leq T-1$.
Suppose that the opposite is true and $a^*_{t+1} \geq a^*_t$ for some $t \leq T-1$ (see Figure 13). Then, by (ii) of Lemma 1, $a'_{t+1}(a^*_{t+1}) \leq a^*_t$. Correspondingly, $V_{t+1}(a'_{t+1}(a^*_{t+1})) \geq V_t(a'_{t+1}(a^*_{t+1}))$.

Therefore,

$$\begin{aligned} V_{t+2}(a^*_{t+1}) &\geq u\left(a^*_{t+1} + y - h^* - \frac{a'_{t+1}(a^*_{t+1})}{1+r}, h^*\right) + \beta V_{t+1}(a'_{t+1}(a^*_{t+1})) \\ &\geq u\left(a^*_{t+1} + y - h^* - \frac{a'_{t+1}(a^*_{t+1})}{1+r}, h^*\right) + \beta V_t(a'_{t+1}(a^*_{t+1})) = V_{t+1}(a^*_{t+1}). \end{aligned}$$

Since $V_{t+2}(a)$ and $V_{t+1}(a)$ have at most one intersection and $V_{t+2}(a) < V_{t+1}(a)$ for sufficiently large a, $V_{t+2}(a) \geq V_{t+1}(a)$ for all $a \leq a^*_{t+1}$. Thus we can inductively apply the above argument and conclude that for any $k \geq 2$ it must be that $V_{t+k}(a^*_{t+1}) \geq V_{t+1}(a^*_{t+1}) > V_\infty(a^*_{t+1})$, which obviously contradicts $\lim_{k\to+\infty} V_{t+k}(a^*_{t+1}) = V_\infty(a^*_{t+1})$.

A symmetric argument is used to prove (ii) of Proposition 2 (namely, it can be verified that $a^*_{t+1} > a^*_t$ as long as $V_{t+1}(a) > \max\{V_\infty(a), V_0(a), V_1(a), ..., V_t(a)\}$ for some $a \geq \underline{a}$). ■

Proof of Proposition 2:

First, recall that when $\beta(1+r) = 1$, the wealth level of the agents staying in h_0 forever does not change over time: $a'_\infty(a) = a$.

Suppose that $V_0(a) > V_\infty(a)$ for sufficiently large a. If $V(a)$ is not differentiable at $a^* > \underline{a}$, it must be that

$$V'_\infty(a^*) = V'(a^{*-}) < V'(a^{*+}) = V'_T(a^*), \tag{12}$$

where T is the optimal waiting time at a^*. But then it is obvious that the consumer with wealth a^* would be strictly better off if, instead of staying at the initial house h_0 forever, he switches to a new house in $T+1$ periods because

$$\begin{aligned} V_{T+1}(a^*) &\geq u(a^* + y - h_0 - \frac{a'_\infty(a^*)}{1+r}, h_0) + \beta V_T(a'_\infty(a^*)) \\ &= u(a^* + y - h_0 - \frac{a^*}{1+r}, h_0) + \beta V_T(a^*) \\ &= u(a^* + y - h_0 - \frac{a^*}{1+r}, h_0) + \beta V_\infty(a^*) = V_\infty(a^*). \end{aligned}$$

The first row uses the fact that $a'_\infty(a)$ is a feasible saving policy for a consumer maximizing $V_{T+1}(a)$, the second is implied by $\beta(1+r) = 1$, and the third follows from the definition of a^* (i.e. $V_\infty(a^*) = V_T(a^*)$). Moreover, (12) implies that the first inequality must be strict. Thus $V_{T+1}(a^*) > V_\infty(a^*)$ and, correspondingly, such a^* cannot be the cutoff wealth level.

A similar argument applies if $V_0(a) < V_\infty(a)$ for sufficiently large a. ■

Proof of Proposition 3:

(i) A standard argument is used to verify that $V_0(a) > V_\infty(a)$ and $V_0(a) > TV_0(a)$ for sufficiently large a when $\Gamma(h_0) = \{h : h \geq h_0\}$ in (5).

To verify that $V_0(a)$ and $V_\infty(a)$ have at most one intersection, we develop a two-steps argument:

Step 1: We can show that for any $a_1 \leq a_2$ the relationship $V_1^*(a_1; h_1) \leq V_1^*(a_2; h_2)$ implies that $V^*(a_1; h_1) \geq V^*(a_2; h_2)$, where $h_1 > h_2$ and $V^*(a; h)$ is the value of staying in house h forever defined in (4) and $V_1^*(a; h)$ is its derivative with respect to the first argument. This property can be established recursively: assume that it holds for some $V^*(a; h_1)$ and $V^*(a; h_2)$ and verify that it carries over to $\mathbb{T}V^*(a; h_1)$ and $\mathbb{T}V^*(a; h_2)$, where

$$\mathbb{T}V^*(a; h) = \max_{a' \geq \underline{a}} \{u(a + y - h - \frac{a'}{1+r}, h) + \beta V^*(a'; h)\}. \tag{13}$$

Suppose that $\mathbb{T}V_1^*(a_1; h_1) \leq \mathbb{T}V_1^*(a_2; h_2)$ for some $a_1 \leq a_2$. For brevity, denote by a_1' and a_2' the optimal saving levels at a_1 and a_2 respectively. By the envelope condition we obtain $u_1(a_1 + y - h_1 - \frac{a_1'}{1+r}, h_1) \leq u_1(a_2 + y - h_2 - \frac{a'}{1+r}, h_2)$. Since $h_1 > h_2$, supermodularity and concavity of $u(\cdot, h)$ imply that

$$c_1 = a_1 + y - h_1 - \frac{a_1'}{1+r} \geq a_2 + y - h_2 - \frac{a_2'}{1+r} = c_2,$$

which generates two useful implications:

$$u(c_1, h_1) > u(c_2, h_2) \tag{14}$$

and

$$a_2' - a_1' \geq (1+r)(\underbrace{a_2 - a_1}_{\geq 0} + \underbrace{h_1 - h_2}_{>0}) > 0. \tag{15}$$

The first order conditions to (13) imply that $u_1(c_1, h_1) \geq \beta(1+r)V_1^*(a_1'; h_1)$ and $u_1(c_2, h_2) = \beta(1+r)V_1^*(a_2'; h_2)$. Note that (15) implies that for the chosen a_1 and a_2 the first order condition to the decision problem defining $\mathbb{T}V^*(a_2'; h_2)$ must hold with equality ($a_2' = \underline{a}$ would imply that $a_1' < \underline{a}$, which is not feasible). Thus $\mathbb{T}V_1^*(a_1; h_1) \leq \mathbb{T}V_1^*(a_2; h_2)$ also implies that $V_1^*(a_1'; h_1) \leq V_1^*(a_2'; h_2)$. By (15) and the assumption of the recursive argument, it follows that $V^*(a_1'; h_1) \geq V^*(a_2'; h_2)$. Combining it with (14) we conclude that $\mathbb{T}V^*(a_1; h_1) > \mathbb{T}V^*(a_2; h_2)$. This concludes the proof of Step 1.

Step 2: We can now use the properties of $V^*(a;h^*)$ and $V_\infty(a) = V^*(a;h_0)$ established in Step 1 to show that $V_0(a)$ and $V_\infty(a)$ cross at most once. If $V_0(a)$ and $V_\infty(a)$ had multiple intersections, there would exist wealth level $\widehat{a}$ in between the two crossing points such that $V_0'(\widehat{a}) = V_\infty'(\widehat{a})$ and $V_0(\widehat{a}) < V_\infty(\widehat{a})$. Thus, similarly to the proof in Lemma 1, it suffices to argue that $V_0'(a) = V_\infty'(a)$ implies that $V_0(a) > V_\infty(a)$.

Consider $a \geq \underline{a}$ such that $V_0'(a) = V_\infty'(a)$. By the envelope condition, $u_1(a+y-\eta-h^*(a)-\frac{a_0'}{1+r}, h^*(a)) = u_1(a+y-h_0-\frac{a_\infty'}{1+r}, h_0)$, where a_0' and a_∞' are the saving rules maximizing $V_0(a)$ and $V_\infty(a)$ respectively, and $h^*(a)$ is the optimal housing level chosen in the decision problem (5). Since $h^*(a) > h_0$, supermodularity implies that $u(a+y-\eta-h^*(a)-\frac{a_0'}{1+r}, h^*(a)) > u(a+y-h_0-\frac{a_\infty'}{1+r}, h_0)$ and, correspondingly, $a_0' < a_\infty'$. From the latter inequality it follows that for the agent maximizing $V_\infty(a)$ the borrowing constraint must be slack. Thus the first order conditions for the agents maximizing $V_0(a)$ and $V_\infty(a)$ are $u_1(a+y-\eta-h^*(a)-\frac{a_0'}{1+r}, h^*(a)) \geq \beta(1+r)V_1^*(a_0', h^*(a))$ and $u_1(a+y-h_0-\frac{a_\infty'}{1+r}, h_0) = \beta(1+r)V_\infty'(a_\infty')$ respectively. Correspondingly, $V_0'(a) = V_\infty'(a)$ also implies that $V_1^*(a_0', h^*(a)) \leq V_\infty'(a_\infty')$. Now we can apply the result established in Step 1 (since $a_0' < a_\infty'$, $V_\infty(a) = V^*(a;h_0)$ and $h_0 < h^*(a)$), concluding that $V^*(a_0', h^*(a)) \geq V_\infty(a_\infty')$ must hold. Therefore, $V_0(a) > V_\infty(a)$ at any $a \geq \underline{a}$ at which $V_0'(a) = V_\infty'(a)$ and, consequently, $V_0(a)$ and $V_\infty(a)$ can have at most one intersection, i.e. (b) of Proposition 1 is satisfied.

To verify that $V_0(a)$ and $TV_0(a)$ have at most one intersection, it is enough to show that $V_0'(a) = TV_0'(a)$ implies that $V_0(a) \geq TV_0(a)$.

Suppose that $V_0'(a) = TV_0'(a)$ at some $a > \underline{a}$. Then from the envelope conditions and supermodularity it follows that $u(a+y-\eta-h^*(a)-\frac{a_0'}{1+r}, h^*(a)) \geq u(a+y-h_0-\frac{a_1'}{1+r}, h_0)$ (where a_0' and a_1' are the optimal saving levels maximizing $V_0(a)$ and $TV_0(a)$ respectively). At the same time, as in Step 2 above, we can verify that for any a such that $V_0'(a) \leq TV_0'(a)$, the inequalities $h^*(a) > h_0$ and $\eta > 0$ imply that $a_0' < a_1'$, and thus the borrowing constraint is slack for the maximizer of $TV_0(a)$. Thus $TV_0'(a) = \beta(1+r)V_0'(a_1')$. Since $\beta(1+r) = 1$ and, by assumption, $V_0'(a) = TV_0'(a)$, it follows that $a_1' = a$ and thus $V_0(a_1') = V_0(a) = V^*(a-\eta, h^*(a))$.

The borrowing constraint in the optimization problem maximizing $V_0(a)$ might be binding, thus the corresponding first order condition guarantees that $V_0'(a) \geq V_1^*(a_0', h^*(a))$. Thus $V_0'(a) = TV_0'(a)$ also implies that $V_1^*(a_0', h^*(a)) \leq V_1^*(a-\eta, h^*(a))$ and, correspondingly, $V^*(a_0', h^*(a)) \geq V^*(a-\eta, h^*(a)) = V_0(a_1')$. Combining it with the inequality for per period utilities obtained above, we conclude that $V_0(a) \geq TV_0(a)$ at any a at which $V_0'(a) = TV_0'(a)$.

Note that when $\beta(1+r) \neq 1$, the wealth level of the consumer maximizing $TV_0(a)$ must adjust (i.e. $a_1' \neq a_1$), and thus the level of housing $h^*(a_1')$ chosen in the following period would be different from $h^*(a)$. In this case, we would no be able to derive any predictions regarding the relative values of $V^*(a_0', h^*(a))$ and $V^*(a_1', h^*(a_1'))$. That is why the argument developed in this proof cannot be used if $\beta(1+r) \neq 1$.

(ii) When $\Gamma(h_0) = \{h : h_0 \geq h > 0\}$, it is straightforward to verify that $V_0(a) < V_\infty(a)$ and $V_0(a) < TV_0(a)$ for sufficiently large a. Thus, as in the proof of part (i), in order to argue that conditions (b) and (c) of Proposition 1 are satisfied, it is sufficient to show that $V_0'(a) = V_\infty'(a)$ implies that $V_0(a) \geq V_\infty(a)$ (and $V_0'(a) = TV_0'(a)$ implies that $V_0(a) \geq TV_0(a)$). However, when $h^*(a) < h_0$, we cannot prove any more that $a_0' < a_\infty'$ (or $a_0' < a_1'$) because $h_0 - h^*(a) - \eta$ can be either positive or negative. Thus in this case we are not able to claim that at a wealth level a at which $V_0'(a) = V_\infty'(a)$ (or $V_0'(a) = TV_0'(a)$) one of the first order conditions necessarily holds with equality. Thus, in the presence of the borrowing constraint, we cannot any more establish that a particular relationship holds for the corresponding continuation values. At the same time, if there are no borrowing constraints, both first order conditions would hold with equality and the proof of (i) trivially extends to the case when the consumers choose a new house from the set $\Gamma(h_0) = \{h : h_0 \geq h > 0\}$. ■