Preferences for Risk in Dynamic Models with Consumption Commitments

Galina Vereshchagina*
Arizona State University

June 2010

Abstract

This paper characterizes the solution to a consumption/savings decision problem in the presence of consumption commitments (goods that involve fixed transaction costs and are infrequently adjusted). Existing studies have suggested that consumption commitments may help explain known anomalies of consumer's risk behavior, such as simultaneous purchase of insurance and lotteries or the equity premium puzzle, because they (i) create incentives for gambling and (ii) amplify risk aversion with respect to moderate risks. This paper argues that even though such predictions naturally arise in static models, they may disappear or even reverse in a dynamic setting. Namely, I show that (i) the possibility of choosing *when* to adjust the consumption level of the commitment good can eliminate the gambling motive and (ii) the agents who plan to make the adjustment in the future become more tolerant to moderate risks than in the environment without consumption commitments.

JEL classification: D14, D91, E21, G12

Keywords: Consumption commitments; Adjustment costs; Dynamic discrete choice; Housing; Risk aversion; Equity premium puzzle; Patience; Borrowing constraints

---

[*] email: Galina.Vereshchagina@asu.edu. I am grateful to the participants of the 2007 NBER Workshop on Consumption Behavior, SED 2007, as well as the participants of the seminars at the University of Iowa, Arizona State University, University of Illinois, New York University and University of California (Santa Barbara) for useful comments and suggestions.

1 Introduction

Many consumption goods, such as housing, land, vehicles and various types of insurance, require substantial expenditures. A distinctive feature of these goods is that adjusting their consumption involves significant transaction costs.[1] As a result, we observe relatively infrequent changes in their consumption level, and it appears as if households commit to buying certain quantities of these goods. That is why literature refers to these types of goods as *consumption commitments*. Since consumption commitments constitute significant share of households' expenses,[2] they have attracted a lot of attention in recent economic literature. In particular, a number of studies have argued that consumption commitments may contribute to explaining two well documented anomalies of risk behavior – simultaneous purchasing of insurance and lotteries as well as high risk aversion with respect to moderate income (or wealth) risks. This paper argues that while these predictions naturally arise in static models, they may disappear or even reverse in dynamic settings. Namely, it shows that the possibility of choosing *when* to adjust the consumption level of the commitment good can eliminate the gambling motive and that the agents, who plan to make the adjustment in the future, become more tolerant to small risks due to the presence of consumption commitments.

To explain why dynamic considerations may have such drastic impact on the effect of consumption commitments on risk attitudes, it is useful to outline the intuition behind the predictions arising in static models first. Consider an agent who can consume two goods, housing (a commitment good) and food (a flexible good, which does not involve any transaction costs). Suppose that *ex ante* the agent commits to a certain level of housing consumption, but *ex post* he receives an income shock and has to decide whether or not to adjust his housing choice. Due to the presence of fixed adjustment costs, small income shocks do not affect the housing choice and, instead, are fully absorbed by the changes in food consumption. In contrast, if there were no housing adjustment costs, the same changes in income would affect consumption levels of both goods. Thus the presence of commitments increases the volatility of consumption of the non-commitment good, and raises the welfare cost of moderate risks. That is why consumption commitments can potentially magnify risk aversion with respect to moderate risks (and hence contribute to explaining the equity premium puzzle). At the same time, if the *ex post* income shock is sufficiently large, a

---

[1] For empirical evidence confirming the presence of transaction costs, see Eberly (1994), Attanasio (2000) and others.

[2] Warren and Tyagi (2003) find that a typical American family earmarks 75% of their income on "fixed expenses" such as mortgage, car payments, etc. Similarly, Chetty and Szeidl (2007) estimate from CES data that around 50% of households' life-time wealth is spent on "commitment goods".

lumpy adjustment of housing consumption occurs. Such discontinuity generates kinks in the indirect utility function, and creates a motive for gambling. That is why consumption commitments may also explain why consumers simultaneously buy lotteries and insurance.

In contrast to static models, in a dynamic environment consumers can choose not only whether or not to switch to a new level of housing consumption, but also *when* to make the adjustment. This gives rise to *transitory* behavior: some consumers decide to remain in their initial house for a while, gradually accumulate (or reduce) their asset holdings, and switch to a new house later on. Such transitory behavior affects consumers' attitudes towards risk in two ways. First, it turns out that the possibility of delaying housing adjustment can eliminate the gambling motive. Formally, I show that, under some conditions, adopting transitory behavior helps consumers to completely smooth out the kinks in their value function. Intuitively, this happens because the choice of *when* to make the discrete adjustment allows the consumer to mitigate the impact of its discreteness since the consumer effectively decides what fraction of the life time to spend with each of the discrete options, thereby averaging his housing consumption over lifetime.[3]

Second, I illustrate that consumption commitments, by giving rise to transitory behavior, may actually lower risk aversion with respect to moderate risks for some consumers. The reason is that consumers who plan to switch to a different house in the future respond to small income shocks by adjusting not only flexible consumption, as in static models, but also asset holdings. As a result, the welfare cost of moderate risks for these agents is not as high as predicted by the static models. In fact, I show that risk attitudes of the consumers who choose to delay housing adjustment do not depend on the curvature of the instantaneous utility function and are determined solely by the relationship between their time discount rate and the interest rate.[4] In particular, when the two rates are equal to

---

[3] In a recent work Stokey (2009) characterizes the portfolio choice in a model with housing adjustment costs in a continuous time environment under the assumptions of CES utility function, Brownian investment returns and no borrowing constraints using the methodology developed in Grossman and Laroque (1990) (and earlier applied by Flavin and Nakagawa (2008) to a model with two goods, only one of which requires adjustment cost). In her model, the resulting value function is also concave and hence the consumers do not have any further gambling motive. However, in Stokey's model the absence of the kinks in the value function may be driven by two separate features: dynamic considerations, as in my paper, and the possibility to invest in risky asset offering a premium over a risk-free bond. It is well known that the presence of the risky assets alone can satisfy consumer's demand for lotteries and thus eliminates non-concavities in the value function. This paper shows that, even in the absence of risky investment opportunities, dynamic considerations alone can help eliminate the gambling motive.

[4] The relationship between the time discount rate and the interest rate determines whether the consumers are risk averse, risk neutral or risk loving. However, the curvature of the resulting value function depends on the shape of the instantaneous utility function (unless the two rates are equal to each other and the consumers are risk neutral).

each other, these consumers become risk neutral with respect to moderate stake risks; their flexible consumption does not change and only their savings adjust if unexpected moderate income shocks occur. Had there been no fixed adjustment costs, all the agents would be risk averse. Thus, in contrast to static models, consumption commitments in a dynamic setup make some of the consumers more risk tolerant by stimulating transiting behavior.

The above observation suggests that the overall effect of consumption commitments on consumers' risk preferences may be ambiguous: fixed adjustment costs reduce the welfare cost of moderate risks for some consumers (those who adopt transitory behavior) but magnify the risk aversion of the others (those who, as in static models, do not plan to make a housing adjustment in the future). Hence, the effect of consumption commitments on 'aggregate risk attitudes' (e.g. aggregate demand for risky assets) depends on how likely the transitions are to occur. In the last section of the paper I develop a series of numerical exercises to analyze whether the transitory behavior could have significant impact on the economy's aggregates. I find that in the benchmark model the likelihood of adopting the transitory behavior after the realization of a permanent income shock is quite large (30% and more). This implies that consumption commitments may be less helpful in explaining the equity premium puzzle than it has been suggested by some of the previous studies.[5,6]

The analysis in this paper directly builds on a recent work by Chetty and Szeidl (2007), who analyze the effects of transaction costs associated with housing adjustments on consumers' risk preferences. They consider an environment where, after the initial housing choice is made, a permanent income shock is realized, and housing adjustments may become desirable. Using this model, the authors analyze how the presence of housing adjustment costs affects the consumers' risk attitudes with respect to further unanticipated income shocks. Even though Chetty and Szeidl's model is dynamic, it is constructed in such a way that all the housing adjustments (if any) are made in the first period only, and the transitive

---

[5] Several other studies, such as Fratantoni (2001) and Fukushima (2005), investigate numerically whether housing commitments magnify the degree of 'aggregate risk aversion' with respect to moderate income risk and find that the effect may indeed be quantitatively significant. These papers, however, rely on very different frictions to generate infrequent housing adjustments: Instead of endogenizing housing commitments through the presence of fixed adjustment costs, they alow housing changes to occur either in certain (exogenously given) time periods or with some (also exogenously given) probability. As a consequence, the possibility of choosing the moment of housing adjustment, is not fully explored in these studies, which significantly limits the role of transitory behavior and may overestimate the degree to which consumption commitments magnify risk aversion.

[6] The findings in this paper are also consistent with the quantitative results by Stokey (2009), who in a roughly calibrated numerical exercise finds that the presence of transaction costs for housing adjustments is not likely to be a significant component in explaining the equity premium puzzle. In some sense, the analysis in this paper provides a theoretical rationale for Stokey's numerical findings.

behavior – which is the main focus of my work – does not arise. In particular, Chetty and Szeidl (2007) assume that the risk-free interest rate is equal to zero and that consumers can borrow against their future income. Under these assumptions, consumers have no incentives to delay housing adjustments.[7] Relaxing either one, however, creates a motive for transitory behavior. Namely, if the interest rate is positive, late switches reduce the present value of the fixed adjustment cost; if borrowing constraints are present, the consumers might need some time to accumulate sufficient funds to finance the switch. Since both of these mechanisms are shut down in Chetty and Szeidl (2007), all housing adjustments in their model happen only in period zero, the transitory behavior never occurs and their results regarding the effects of housing adjustment costs on consumers' risk attitudes are similar to the predictions obtained in static models. I extend Chetty and Szeidl's environment by allowing for a positive interest rate and/or borrowing constraints, which enables me to analyze the effects of transitory behavior.

In addition to analyzing the effects of consumption commitments on risk preferences in dynamic settings, this paper also describes a novel relationship between the patience of the consumers exhibiting transitory behavior and their risk attitudes. As mentioned above, I find that the relationship between the time discount rate and the risk-free interest rate is crucial for shaping the risk preferences of such consumers: when the two rates are equal to each other, these consumers are risk neutral with respect to small income (or wealth) shocks; and when these rates are different, the consumers are either risk averse or risk lovers, depending on whether they are saving or dissaving prior to the switch. In particular, the less patient is the consumer who is accumulating wealth in order to make an adjustment in the future, or the smaller is the risk-free interest rate paid on his savings – the more risk tolerant he turns out to be. While the relationship between the risk-free interest rate and risk tolerance is very intuitive,[8] the relationship between patience and risk aversion is less plausible. The driving force behind it is that, for the consumers exhibiting transitory behavior, small income (or wealth) shocks lead to variations in the timing of a future discrete housing adjustment. This discrete adjustment generates a discrete increase in the instantaneous utility function. Time-separability and exponential discounting in the lifetime utility create convexity in consumer's

---

[7] This is true if the time discount rate and the interest rate are equal to each other. Such assumption is made in Chetty and Szeidl (2007) and it considerably simplifies the analysis. Since in their paper the interest rate is set to zero, the time discount rate is also zero, which implies that Chetty and Szeidl's model must be formulated in finite horizon.

[8] Investing in a safe asset is an outside option to taking risk (investing in a risky asset); thus higher risk-free rate reduces the willingness to opt out for a risky option.

preferences with respect to the timing of the discrete adjustment.[9,10] The smaller is the time discount rate, the bigger is the degree of this convexity, and the more risk-tolerant is the consumer. Thus, among the consumers who save in order to implement the discrete adjustment in the future, the less patient ones also turn out to be more risk-tolerant.

Another novel application of the findings in this paper pertains to the effect of borrowing constraints on risk attitudes. Recall that in an environment without consumption commitments (e.g. Aiyagari (1994)), borrowing constraints magnify the cost of uninsured income risk because consumption of constrained agents is more sensitive to negative income shocks than consumption of unconstrained agents. In the models with fixed adjustment costs, however, borrowing constraints may increase the incidence of transitory behavior, for instance because the adjustment cost must be paid upfront. Since, as discussed above, the agents adopting transitory behavior exhibit lower risk aversion than those who make an adjustment immediately, this paper provides an example of an economic environment in which, contrary to conventional wisdom, borrowing constraints can make some consumers more risk tolerant. This is an interesting observation, which, to my knowledge, has not been widely discussed in the previous literature, and which may have interesting implications, for example, for the effects of borrowing limits on portfolio choice.[11]

Finally, this paper also makes a methodological contribution. To describe housing behavior in the presence of fixed adjustment costs, I need to solve a dynamic model of discrete choice. It is well known that, in general, models of this type are hard to characterize analytically. In particular, it is difficult to establish single crossing of the value functions associated with the available discrete options (such as remaining in the initial house forever versus transiting / switching to a new one)).[12] One sufficient condition that would guarantee such single crossing (and which is often used in dynamic discrete choice models[13]) is the monotonicity of the difference between two value functions: the value of switching to the new

---

[9] If a utility gain of size Δu occurs at time t, its present value at time 0 is measured by $\beta^t \Delta u$ ($\beta \in (0,1)$ in discrete time) or $\exp(-\rho t)\Delta u$ ($\rho > 0$ in continuous time), which is convex in t.

[10] Note, however, that such convexity with respect to the timing of the adjustment does not imply that the consumer must be risk lover with respect to income (or wealth) shocks. This is because randomizing over income (or wealth) may, depending on the size of the the risk-free interest rate, increase or reduce the expected time needed to accumulate the amount of wealth at which the adjustment becomes optimal.

[11] It has been noted that in the models of discrete occupational choice borrowing constraints may generate kinks in the agents' indirect utility function, thereby creating gambling motives. For example, Vereshchagina and Hopenhayn (2009) argue that this feature can contribute to explaining excessive entrepreneurial risk taking observed in the data.

[12] Establishing single crossing property allows to derive the cutoff rules for the state variable which characterize the agents' discrete choice.

[13] See, for example, Dixit and Pindyck (1993).

option in the current period and the value of switching to the same option in the following period. Unfortunately, this condition does not hold in many discrete choice models where the instantaneous payoff is bounded from above – as it is typical in the models of consumer choice. I show that in this class of models standard recursive methods can be used to derive a much weaker condition guaranteeing single crossing. This sufficient condition can be easily verified not only in the model with housing adjustment costs studied in this paper but also in other similar environments with perfect foresight, such as occupational choice[14], costly technology adoption[15], endogenous default or endogenous retirement models. Hence, the results derived in this paper can also be immediately applied to other economic problems. For instance, they can generate a number of testable predictions about the behavior of the consumers who make savings to start their own business, upgrade to a new technology, go to school, retire, etc.

The rest of the paper is organized as follows. Section 2 describes the modeling environment. Section 3 discusses the effects of consumption commitments on risk attitudes in a restricted version of the model in which, as in Chetty and Szeidl (2007), housing adjustments can only happen in period 0. Section 4 describes the solution approach and discusses its methodological contribution. Section 5 presents the main results of the paper regarding the effects of transitory behavior on consumers' risk preferences and section 6 applies them to the housing model in question. Finally, section 7 develops a series of numerical exercises and argues that the impact of the transitory behavior on aggregate risk attitudes is likely to be quantitatively significant.

2 The Environment

Consider an agent who receives utility from two consumption goods, food and housing. The agent's life-time utility is given by:

$$\sum_{t=0}^{+\infty} \beta^t u(c_t, h_t),$$

---

[14] Occupational choice models with uncertainty are heavily used in macroeconomic models. The examples include Lucas (1974), Quadrini (2000), Bohacek (2006), Cagetti and DeNardi (2006) and others. The examples of occupational choice models with perfect foresight are studied in Buera (2006) and Vereshchagina and Hopenhayn (2009).

[15] See, for instance, Greenwood and Jovanovic (1990) and Khan and Ravikumar (2002).

where $\beta \in (0,1)$ is the time discount factor, and the instantaneous utility in period t is derived from the flow consumption of food c_t and housing h_t. Assume that $u(\cdot,\cdot)$ is defined on $\mathbb{R}^2_+$, is bounded from above, strictly increasing, strictly concave, and satisfies Inada conditions.

Suppose that in every period the agent receives income $y \geq 0$, which can be spent on consumption goods or stored in a risk-free asset that offers interest rate r. In much of the analysis below I assume that $\beta(1+r) = 1$, though many of the results, including the effects of transitory behavior on risk attitudes, are derived for any $\beta(1+r) > 0$. The consumer's total asset holdings a_t might be subject to the borrowing constraint $a_t \geq \underline{a}$ (if $\underline{a} = -\infty$ there are no borrowing limits and the transversality condition $\lim_{t\to+\infty} \frac{a_t}{(1+r)^t} = 0$ is imposed instead). Denote the agent' initial asset holdings by a_0.

Food consumption c_t is flexible and can be adjusted at no cost. In contrast, a fixed adjustment cost $\eta > 0$ must be paid if the agent switches from one housing level to another. The presence of the transaction cost η guarantees that the housing adjustments, if they occur, are discrete. At this stage, I assume that the agent's initial housing consumption h_0 is exogenously given.[16] This assumption is relaxed in numerical exercises in section 7. One way of endogenizing h_0 is to introduce permanent income shocks at $t = 0$ and allow consumers to choose h_0 optimally prior to the realization of uncertainty in y. Under such interpretation, most of the paper, with the exception of section 7, is devoted to the analysis of the consumer's problem after the initial housing decision has already been made and all uncertainty has been resolved.

Notice that, since $\beta(1+r) = 1$ and there is no uncertainty, the consumer makes at most one housing adjustment over the course of lifetime, after which he will maintain constant consumption levels of food and housing for the rest of his life. This considerably simplifies the characterization of the consumer's optimal choice. Namely, the decision problem of the consumer with initial housing commitment h_0, initial wealth a_0 and per period income y can be written as the choice of the food consumption and wealth profiles $\{c_t\}_{t=0}^{+\infty}$ and $\{a_t\}_{t=1}^{+\infty}$, the moment of housing adjustment T as well as the level of housing consumption h^* to which the consumer switches in period N:

---

[16] Since h_0 is fixed, the adjustment cost η may be interpreted as a function of h_0, i.e. a fraction of the total housing value.

$$
\begin{aligned}
V(a_0,h_0) = \max_{\{c_t,a_t\},h^*,T\in\{0,1,2,\ldots,+\infty\}} & \sum_{t=0}^{T-1}\beta^t u(c_t,h_0) + \sum_{t=T}^{+\infty}\beta^t u(c_t,h^*) \\
\text{s.t. } & c_t + h_0 + \frac{a_{t+1}}{1+r} \le a_t + y, \quad 0 \le t < T, \\
& c_t + h^* + \eta + \frac{a_{t+1}}{1+r} \le a_t + y, \quad t = T, \\
& c_t + h^* + \frac{a_{t+1}}{1+r} \le a_t + y, \quad t > T, \\
& a_t \ge \underline{a}, \quad t \ge 0, \\
& a_0 \text{ and } h_0 \text{ are given.}
\end{aligned} \tag{1}
$$

Note that the agent might decide to switch to a new house right away by setting $T = 0$ (in which case the first set of the budget constraints is irrelevant) or to remain in his initial house h_0 forever by setting $T = +\infty$.

Decision problem (1) is an extension of the problem studied in Chetty and Szeidl (2007). The environment in this paper differs from Chetty and Szeidl (2007) in two major respects: here the interest rate r is allowed to be positive and the borrowing constraints might be present. Either of these features might induce the consumer to wait for a while prior to adjusting housing consumption, either in order to decrease the present value of the adjustment cost η (if $r > 0$) or in order to accumulate enough funds to finance the switch (which might be relevant when the borrowing constraints are binding). In contrast, Chetty and Szeidl (2007) analyze the model with $r = 0$ and without the borrowing constraints.[17] That is why in their environment the consumers make housing adjustments (if any) only in period 0.

Note also that, following Chetty and Szeidl (2007), I study a dynamic perfect foresight model. On the one hand, discussing the implications for consumers' risk attitudes in the model in which there is no uncertainty may sound unusual. On the other hand, such approach is quite instructive because the analysis of the shape of the indirect utility function in the deterministic environment indicates whether the consumers would be willing to undertake risk (i.e. invest in risky assets) if such option was available. Alternatively, from the solution of this deterministic problem one can immediately derive the welfare losses associated with unexpected permanent income (or wealth) shocks. In this sense, by saying that consumption commitments make the consumer less (more) risk averse, I (as well as Chetty and Szeidl

---

[17] Similar to this paper, Chetty and Szeidl (2007) also assume that $\beta(1 + r) = 1$ which eliminates the incentives for repeated housing adjustments. Since they set $r = 0$, $\beta(1 + r) = 1$ implies that $\beta = 1$. Consequently, their model can only be formulated in a finite horizon. In contrast, the model in this paper is formulated in an infinite time horizon.

(2007)) imply that the welfare losses associated with unexpected permanent socks are smaller (higher) in the environment with consumption commitments than without.

3 A restricted model: no housing adjustments after $t = 0$ and no borrowing constraints

As a benchmark, it is useful to briefly discuss the solution to the consumer's decision problem under the assumption that delaying housing adjustments is not allowed and there are no borrowing limits (i.e. $\underline{a} = -\infty$). Such modification provides a useful comparison with the results of Chetty and Szeidl (2007), as well as the impact of consumption commitments on risk attitudes in static models. Under these restrictions, the consumer chooses between staying in his initial house forever and switching to a new house of optimal size h^* immediately. Denote the value of staying in the initial house h_0 by $V_\infty(a)$. It can be expressed recursively as

$$V_\infty(a) = \max_{a' \geq \underline{a}} \{u(a + y - h_0 - \frac{a'}{1+r}, h_0) + \beta V_\infty(a')\}. \tag{2}$$

If the consumer could adjust housing consumption costlessly, his value $\widetilde{V}(a)$ would be

$$\widetilde{V}(a) = \max_{h^*} \{V^*(a; h^*)\}, \tag{3}$$

where $V^*(a; h^*)$ is the value of staying in house h^* forever:

$$V^*(a; h^*) = \max_{a' \geq \underline{a}} \{u(a + y - h^* - \frac{a'}{1+r}, h^*) + \beta V^*(a'; h^*)\}. \tag{4}$$

Obviously, $V_\infty(a) = V^*(a; h_0)$.

Since the consumer has to pay cost η for housing adjustments, the value $V_0(a)$ of switching to a new house immediately can be found as

$$V_0(a) = \max_{a' \geq \underline{a}, h^*} \{u(a + y - h^* - \eta - \frac{a'}{1+r}, h^*) + \beta V^*(a'; h^*)\}. \tag{5}$$

Note also that $V_0(a) = \widetilde{V}(a - \eta)$.

The consumer's value is then given by $\max\{V_\infty(a), V_0(a)\}$. The value functions $\widetilde{V}(a)$, $V_\infty(a)$ and $V_0(a)$ are illustrated on Figure 1. Observe that $\widetilde{V}(a) \geq V_\infty(a)$ for all a, and the values $\widetilde{V}(a)$ and $V_\infty(a)$ are tangent at the wealth level $\widetilde{a}$ at which housing consumption

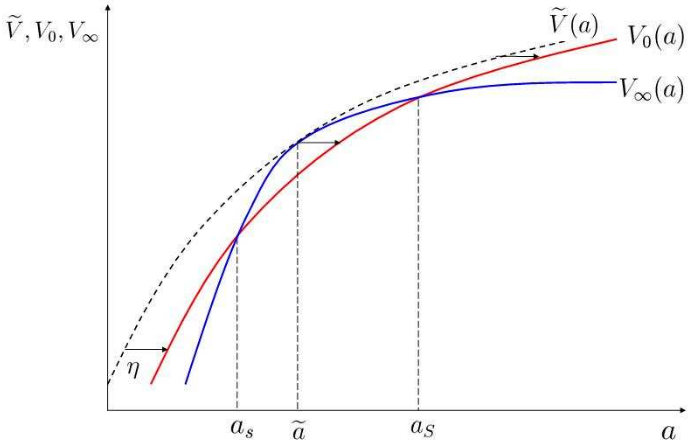

Figure 1: An (s,S) property of the consumer's housing choice when housing adjustments are made only at $t = 0$.

h_0 would be chosen had there been no adjustment costs. Since $\widetilde{V}(a)$ is strictly increasing and $V_0(a)$ is obtained by shifting $\widetilde{V}(a)$ to the right by η, $V_0(\widetilde{a}) < V_\infty(\widetilde{a})$, implying that the consumer would choose not to make housing adjustments in the neighborhood of $\widetilde{a}$. Later on (in Proposition 3) I will verify that $V_0(a)$ intersects $V_\infty(a)$ twice, at $a_s < \widetilde{a}$ and $a_S > \widetilde{a}$, and that $\max\{V_\infty(a), V_0(a)\}$ is not smooth at these intersection points. Thus the consumer's housing decision has an (s, S) property: he stays in the initial house if $a \in [a_s, a_S]$, moves into a smaller house if $a < a_s$ and moves into a bigger house if $a > a_S$.

The shape of the resulting value function $\max\{V_\infty(a), V_0(a)\}$ describes the agent's risk preferences because it determines the welfare costs of unexpected permanent wealth (or income, since there are no borrowing limits) shocks. Since this value function has kinks at a_s and a_S, the agent has a gambling motive in some wealth ranges. At the same time, inside the (a_s, a_S) interval, the agent's value function appears to have higher curvature[18] than $\widetilde{V}(a)$. This is obviously true at $\widetilde{a}$, and Chetty and Szeidl (2007) show that, under some assumptions on the instantaneous utility function $u(c, h)$[19], this also holds for any other wealth level inside (a_s, a_S). The implication is that the presence of housing adjustment costs magnifies the degree of risk aversion in the (s, S) band.

In what follows I show that if the agent is allowed to delay housing adjustments, his value function changes in two ways. First, it becomes globally concave and the gambling

---

[18] A measure of curvature of a value function $v(a)$ that is used in Chetty and Szeidl (2007) to evaluate risk preferences is the the coefficient of relative risk aversion (CRRA) $\gamma(a) = v''(a)a/v'(a)$.

[19] Namely, Chetty and Szeidl (2007) require that $u(c, h)$ is either homogeneous of some degree or is separable in two goods and has constant CRRA in food consumption.

motives disappear. Second, the interval, within which the agent's risk aversion is magnified relative to the environment without adjustment costs, shrinks, and, instead, there appear the intervals where the housing adjustment costs make the consumer *more risk tolerant*. In the last section of the paper, I illustrate in a series of quantitative exercises that such intervals are likely to be large relative to the intervals where the risk aversion is magnified, and hence their presence may have a significant impact on the risk aversion at the 'aggregate level'.

4 Solution to the full dynamic problem: recursive approach

This section develops a recursive approach used to solve decision problem (1). It is important to point out that this solution method is not specific to the housing model set up earlier and can be applied to a broader class of dynamic discrete choice models with perfect foresight, such as models of occupational choice, technology adoption, retirement, etc. This observation might make the methodology developed below attractive to a broader set of readers, particularly those working with similar dynamic models of discrete choice.

It is convenient to explicitly reformulate the consumer's decision problem (1) as the choice of the moment of switching to a new house. For brevity, drop h_0 and denote by $V(a)$ the value of the consumer who has not moved to a new house yet. It can be represented as

$$V(a) = \max\{V_\infty(a), V_0(a), V_1(a), V_2(a), V_3(a), ...\}, \tag{6}$$

where $V_t(a)$ is the value of the consumer who plans to make a housing adjustment in t periods. Given $V_0(a)$, the sequence of the value functions $\{V_t(a)\}_{t=1}^{+\infty}$ can be determined recursively:

$$V_{t+1}(a) = \max_{a' \geq \underline{a}} \{u(a + y - h_0 - \frac{a'}{1+r}, h_0) + \beta V_t(a')\} = \mathbb{T}V_t(a), \quad t \geq 0. \tag{7}$$

Equation (7) says that the consumer, who plans to move into a new house in $t+1$ periods, chooses his savings optimally and in the next period continues with the value of the consumer who plans to move into a new house in t periods. Obviously, the value $V_\infty(a)$ of the consumer who remains in house h_0 forever (defined in (8)) is the fixed point of the operator $\mathbb{T}$:

$$V_\infty(a) = \lim_{t \to +\infty} \mathbb{T}^t V_0(a). \tag{8}$$

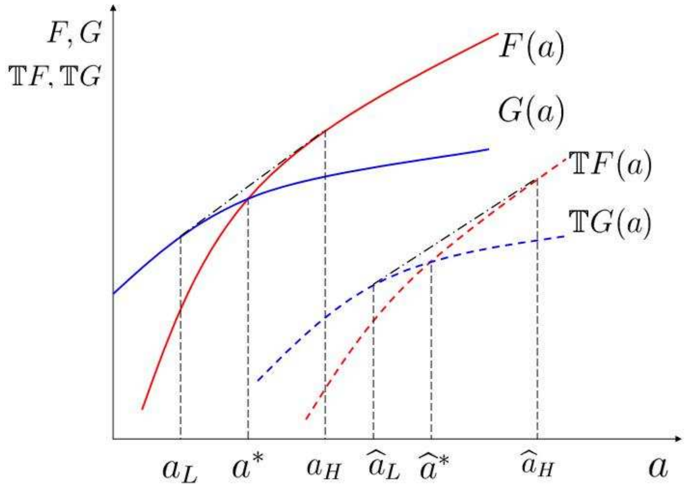

Figure 2: Illustration to Lemma 1

Such recursive representation helps to fully characterize the solution to the consumer's decision problem (1) and describe the properties of the value function $V(a)$. The crucial step in characterizing $V(a)$ is establishing the following Lemma:

Lemma 1 *Let $\mathcal{C}([\underline{a}, +\infty))$ be a set of bounded, strictly increasing, strictly concave and continuously differentiable functions on $[\underline{a}, +\infty)$, and let $\mathbb{T} : \mathcal{C}([\underline{a}, +\infty)) \to \mathcal{C}([\underline{a}, +\infty))$ be an operator defined in (7). Suppose that $F \in \mathcal{C}([\underline{a}, +\infty))$ and $G \in \mathcal{C}([\underline{a}, +\infty))$ have at most one intersection and $F(a) > G(a)$ for all $a > a^* \geq \underline{a}$ (see Figure 2). Then*

- *(i) $\mathbb{T}F(a)$ and $\mathbb{T}G(a)$ cannot have more than one intersection and $\mathbb{T}F(a) > \mathbb{T}G(a)$ for sufficiently large a; let $\widehat{a}^* = \min\{a \geq \underline{a} : \mathbb{T}F(a) \geq \mathbb{T}G(a)\}$;*
- *(ii) $a'_F(a) \geq a^*$ for all $a \geq \widehat{a}^*$ and $a'_G(a) \leq a^*$ for all $a < \widehat{a}^*$, where $a'_F(a)$ and $a'_G(a)$ are the optimal saving policy rules of the agents maximizing $\mathbb{T}F(a)$and $\mathbb{T}G(a)$ respectively;*
- *(iii) denote by a_L and $a_H \geq a_L$ the wealth levels at which $F(a)$ and $G(a)$ are tangent with their common tangent line, and by $\widehat{a}_L$ and $\widehat{a}_H$ the corresponding wealth levels for the pair of functions $\mathbb{T}F(a)$ and $\mathbb{T}G(a)$. Suppose that $a_L > \underline{a}$ and $\widehat{a}_L > \underline{a}$. If $a'_G(\widehat{a}_L)$ and $a'_F(\widehat{a}_H)$ are interior then $a'_G(\widehat{a}_L) = a_L$ and $a'_F(\widehat{a}_H) = a_H$.*

Remark 1 *Lemma 1 holds for any value of $\beta(1 + r) > 0$ (not only $\beta(1 + r) = 1$).*

The results of Lemma 1 are very intuitive. The first statement says that if the choice between the two options $F(a)$ and $G(a)$ is described by a unique threshold rule then the decision problem of the agent who chooses whether to continue with $F(a)$ or $G(a)$ in the future is also described by the unique threshold rule for the agent's current wealth level. The second statement says that the savings policy of the agent is consistent with his future choice between $F(a)$ and $G(a)$: for instance, if the agent prefers $\mathbb{T}F(a)$ in the current period then he would save so much that in the next period he would indeed prefer to choose option $F(\cdot)$. The last statement follows directly from envelope and first order conditions and implies that the agent's optimal saving policy maps the tangent points of the current period value function(corresponding to different options) with their common tangent line into the points on the future value functions with similar property. The proof of Lemma 1 is in the Appendix.

Note that Lemma 1 does not require the difference between $F(a)$ and $G(a)$ to be monotone in a. In contrast, such assumption is commonly made in dynamic discrete choice literature to establish single crossing of $\mathbb{T}F(a)$ and $\mathbb{T}G(a)$ (e.g. Dixit and Pindyck (1993) or Greenwood and Jovanovic (1990)). It is indeed straightforward to verify that, given the structure of the operator $\mathbb{T}$, if $F'(a) \geq G'(a)$ were satisfied for all a then $\mathbb{T}F'(a) \geq \mathbb{T}G'(a)$ would also hold for all a (implying their single crossing).[20] However, when $F(a)$ and $G(a)$ represent the agent's life-time values, they naturally inherit the properties of the instantaneous utility function, and, if the latter is bounded from above, it would follow that $\lim_{a\to+\infty} F(a) - G(a) = 0$. Therefore, since $F(a^*) - G(a^*) = 0$ and $F(a^*) - G(a^*) > 0$ for $a > a^*$, the difference $F(a^*) - G(a^*)$ cannot be monotone. Thus in the model studied in this paper, as well as in other models where instantaneous payoff is bounded from above[21], the differences between the two value functions associated with different options would not be monotone. Hence, the technique used in the existing literature to establsh single crossing would not apply in these models. Lemma 1 shows that single crossing of $\mathbb{T}F(a)$ and $\mathbb{T}G(a)$ can still be established in these environments, as long as $F(a)$ and $G(a)$ have at most one intersection (a weaker condition than monotonicity of their differences). It is also important to point out that the proof of Lemma 1 is quite general, in the sense that it does not rely on the structure of the particular housing model, and uses only standard recursive arguments. Thus, in my opinion, the results of Lemma 1 may be of particular interest to the broader set of readers working with dynamic models of discrete choice.

Lemma 1 can now be used to provide a complete characterization of $V(a)$. Suppose that the choice between never making an adjustment (choosing $V_\infty(a)$) and making it immediately

---

[20] This implication follows directly from the envelope and first order conditions.

[21] Such assumption is often made in macroeconomic models for technical reasons and holds for commonly used utility functions, such as constant relative risk aversion $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma > 1$ or exponential $u(c) = 1 - e^{-ac}$ with $a > 0$.

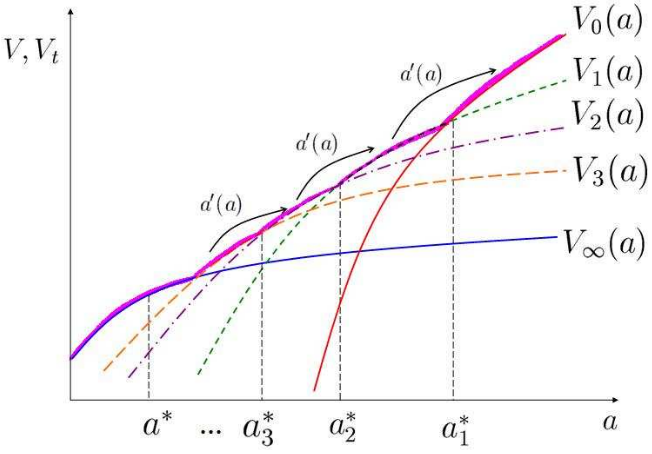

Figure 3: An example of the function $V(a) = \max\{V_\infty(a), V_0(a), V_1(a), V_2(a), ...\}$ and the optimal savings policy $a'(a)$ when $V_\infty(a) < V_0(a)$ for sufficiently large a.

(choosing $V_0(a)$) is characterized by a single cutoff level. It is then convenient to distinguish between two cases: when $V_0(a) > V_\infty(a)$ for large a and when $V_0(a) < V_\infty(a)$ for large a (illustrated on Figures 3 and 4 respectively).[22] Consider, for instance, the first case shown on Figure 3: if adjustment was possible only in period 0 (as in section 3), relatively poor agents would keep the initial option, and the rich ones would make an adjustment immediately. Lemma 1 is used to formally show that the agents with the intermediate wealth levels would choose to postpone making adjustment, with the waiting time declining in agents' initial asset holdings. Namely, there exists a monotonically declining sequence of cutoff wealth levels $\{a_t^*\}_{t=1}^T$ (with T possibly infinite) such that the agents with initial wealth in $(a_{t+1}^*, a_t^*]$ choose to wait for exactly t periods and adopt a saving policy that places them into a wealth interval $(a_t^*, a_{t-1}^*]$ in the following period. Graphically, the value functions from the sequence $\{V_t\}_{t=1}^\infty$ gradually 'move' from $V_0(a)$ to $V_\infty(a)$ helping the agent to smooth out the kink in $\max\{V_\infty(a), V_0(a)\}$.

The first statement of Lemma 1 is used to show that if $V_0(a)$ and $V_1(a)$ have at most one intersection then any two consecutive functions $V_t(a)$ and $V_{t+1}(a)$ from the sequence $\{V_t(a)\}_{t=1}^{+\infty}$ defined by (7) also have at most one intersection. The second statement of Lemma 1 is used to describe the evolution of consumer's asset holdings, and its last statement helps to characterize the shape of $V(a)$ at the intermediate wealth levels (namely, its convex / concave properties), and hence to describe the risk attitudes of the consumers planning to

---

[22] As shown in section 6 later, in the housing model set up earlier these two cases correspond to limiting the agent to only switching to a bigger house or only switching to a smaller house. The choice between whether to switch to a bigger or a smaller house is then characterized separately.

make an adjustment in the future. The formal characterization of $V(a)$ and the optimal savings policy is provided by the Proposition below:

PROPOSITION 1 *Let $\mathcal{C}([\underline{a}, +\infty))$ be a set of bounded, strictly increasing, strictly concave and continuously differentiable functions on $[\underline{a}, +\infty)$, let $\mathbb{T} : \mathcal{C}([\underline{a}, +\infty)) \to \mathcal{C}([\underline{a}, +\infty))$ be an operator defined in (7), and let V_∞ be the fixed point of this operator. Suppose that $V_0 \in \mathcal{C}([\underline{a}, +\infty))$ satisfies the following conditions:*

(a) $V_0(a)$ and $\mathbb{T}V_0(a)$ have a unique intersection at $a_1^* \geq \underline{a}$, i.e. $V_0(a_1^*) = \mathbb{T}V_0(a_1^*)$;

(b) $V_0(a)$ and $V_\infty(a)$ have at most one intersection.

Then $V(a)$ defined in (6) has the following properties:

(i) if $V_\infty(a) < V_0(a)$ for sufficiently large a (see Figure 3) then there exists $a^* \leq a_1^*$ and a strictly decreasing (possibly empty or infinite) sequence of the cutoff levels $\{a_t^*\}_{t=2}^T$ with $a^* \leq a_t^* \leq a_1^*$, $t = \overline{2, T}$ such that

$$V(a) = \begin{cases} V_\infty(a), & a \in [\underline{a}, a^*] \\ V_T(a), & a \in (a^*, a_T^*] \\ V_{t-1}(a), & a \in (a_t^*, a_{t-1}^*], \quad t = \overline{2, T} \\ V_0(a), & a \in (a_1^*, +\infty) \end{cases}$$

Additionally, $a'(a) \in (a_t^*, a_{t-1}^*]$ for all $a \in (a_{t+1}^*, a_t^*]$, $t = \overline{1, T}$, where $a'(a)$ is the optimal savings policy (here $a_{T+1}^* = a^*$).

(ii) if $V_\infty(a) > V_0(a)$ for sufficiently large a (see Figure 4) then there exists $a^* \geq a_1^*$ and a strictly increasing (possibly empty or infinite) sequence of the cutoff levels $\{a_t^*\}_{t=2}^T$ with $a_1^* \leq a_t^* \leq a^*$, $t = \overline{2, T}$ such that

$$V(a) = \begin{cases} V_0(a), & a \in [\underline{a}, a_1^*] \\ V_t(a), & a \in (a_t^*, a_{t+1}^*], \quad t = \overline{1, T-1} \\ V_T(a), & a \in (a_T^*, a^*] \\ V_\infty(a), & a \in (a^*, +\infty) \end{cases}$$

Additionally, $a'(a) \in (a_{t-1}^*, a_t^*]$ for all $a \in (a_t^*, a_{t+1}^*]$, $t = \overline{1, T}$, where $a'(a)$ is the optimal savings policy (here $a_{T+1}^* = a^*$).

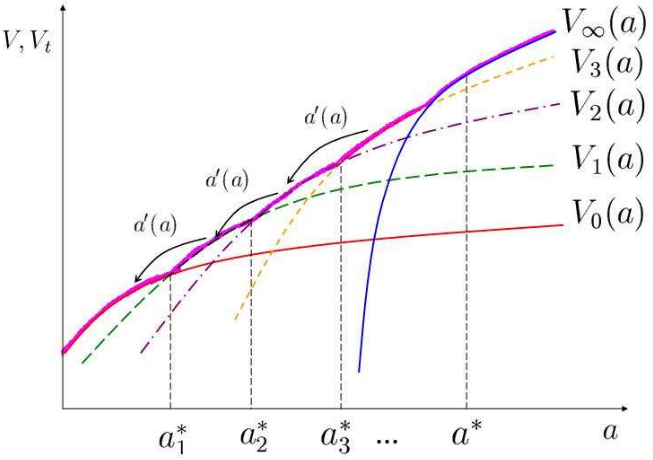

Figure 4: An example of the function $V(a) = \max\{V_\infty(a), V_0(a), V_1(a), V_2(a), ...\}$ and the optimal savings policy $a'(a)$ when $V_\infty(a) > V_0(a)$ for sufficiently large a.

The proof of Proposition 1 is in the Appendix. It follows from Proposition 1 that an agent's housing decision is described by a set of simple cutoff rules: in case (i) the agent never makes an adjustment if his initial wealth is sufficiently small (*'inaction'* region $[\underline{a}, a^*]$), makes an adjustment right away if he is sufficiently rich (*'immediate adjustment'* interval $[a_1^*, +\infty)$) and delays making an adjustment if his wealth falls into the intermediate interval (a^*, a_1^*) (*'transitory'* interval). In addition, Proposition 1 also establishes that within the transitory interval the consumer's wealth monotonically adjusts over time, and thus the closer the agent's initial wealth is to a^*, the longer he would remain with his initial option.

Figures 3 and 4 show only the first three elements of the sequence of value functions $\{V_t\}_{t=0}^\infty$ and indicate that potentially there may be more cutoff levels between a^* and a_3^*. In general, the maximum waiting time T does not have to be finite. In fact, Proposition 2 in the following section establishes that if $\beta(1+r) = 1$ then $T = +\infty$ as long as $\max\{V_\infty(a), V_0(a)\}$ is not globally concave.

Finally, it is important to point out that the characterization of $V(a)$ in Proposition 1 holds for any value of $\beta(1 + r)$ as long as conditions (a) and (b) are satisfied:

Remark 2 *Proposition 1 holds for any value of* $\beta(1 + r) > 0$ *(not only* $\beta(1 + r) = 1$*).*

This observation gives the possibility to analyze the relationship between the consumer's patience and his risk preferences in the transitory interval, one of the questions addressed in the following section.

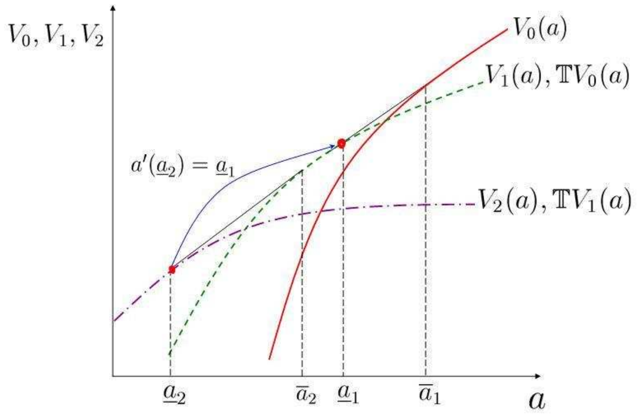

Figure 5: Mutual allocation of $V_0(a)$, $V_1(a)$ and $V_2(a)$ in case (i) of Proposition 1. The relation between $\overline{a}_2$ and $\underline{a}_1$ depends on the value of $\beta(1 + r)$.

5 The effects of transitory behavior on risk attitudes

This section provides a more detailed characterization of the agent's value function in the transitory intervals and discusses its implication for risk preferences of the consumers adopting transitory behavior. I first describe risk attitudes of the agents who are *transiting upwards* – those who are in the process of accumulating wealth in order to switch to a new option in the future (i.e. statement (i) or Proposition 1 applies). While doing this, I distinguish between different values of $\beta(1 + r)$ since it turns out that these consumers' risk attitudes crucially depend on $\beta(1 + r)$. Then I discuss how the predictions change for the consumers *transiting downwards* – those who are gradually consuming their wealth out, while planning to make a discrete adjustment later on (i.e. statement (ii) or Proposition 1 applies).

5.1 Upward transition, $\beta(1 + r) = 1$

Suppose that $V_0(a)$ and $V_\infty(a)$ satisfy all conditions of Proposition 1 and that $V_0(a) > V_\infty(a)$ for sufficiently large a, i.e. statement (i) of Proposition 1 applies. The goal of this section is to characterize the curvature of the value function in the transitory interval (a_*, a_1^*). In order to do this, I next describe in more details mutual allocation of the value functions from the sequence $\{V_t(a)\}_{t=0}^{T}$.

Denote by $\underline{a}_1$ and $\overline{a}_1 > \underline{a}_1$ the wealth levels at which $V_1(a)$ and $V_0(a)$ meet their common tangent line (see Figure 5),[23] and by $\underline{a}_2$ and $\overline{a}_2 > \underline{a}_2$ the wealth levels at which $V_2(a)$ and

---

[23] Note that $\overline{a}_1 > \underline{a}_1$ is the only interesting case because $\overline{a}_1 = \underline{a}_1$, in conjunction with $\beta(1 + r) = 1$ would imply that $V_t(\overline{a}_1) = V_0(\overline{a}_1)$ and $V_t'(\overline{a}_1) = V_0'(\overline{a}_1)$ for all $t \geq 1$. Then a limiting argument suggests that

$V_1(a)$ meet their common tangent line. When all these tangent points are interior, (iii) of Lemma 1, in conjunction with $V_2(a) = \mathbb{T}V_1(a)$ and $V_1(a) = \mathbb{T}V_0(a)$, implies that $a'(\underline{a}_2) = \underline{a}_1$. Thus, the first order and envelope conditions imply that

$$V_2'(\underline{a}_2) = \beta(1+r)V_1'(\underline{a}_1). \tag{9}$$

At the same time, by definition of $\underline{a}_2$ and $\overline{a}_2$, and since $\underline{a}_2$ is interior, it must be true that

$$V_2'(\underline{a}_2) = V_1'(\overline{a}_2). \tag{10}$$

When $\beta(1+r) = 1$, the combination of (9) and (10) implies that $V_1'(\underline{a}_1) = V_1'(\overline{a}_2)$. Since $V_1(a)$ is strictly concave, it follows that $\underline{a}_1 = \overline{a}_2$. Therefore, $V_0(a)$, $V_1(a)$ and $V_2(a)$ have a common tangent line.

The same argument can be applied inductively to all the value functions from the sequence $\{V_t(a)\}_{t=1}^T$, which implies that all of them have one common tangent line, and if the agent's initial wealth a_0 coincides with one of the tangent points, then his continuation wealth profile would include all other tangent points to the right of a_0 (see Figure 6).[24] Moreover, if a^* is interior, (iii) of Lemma 1 can be also applied to a pair of functions $V_\infty(a) = \mathbb{T}V_\infty(a)$ and $V_1(a) = \mathbb{T}V_0(a)$ implying that $V_\infty(a)$ is also tangent to the common tangent line of $\{V_t(a)\}_{t=1}^T$.[25] In addition, it can be verified that this tangent point occurs at exactly a^*. These observations are summarized in the Proposition below.

PROPOSITION 2 *(properties of $V_0(a)$ under $\beta(1+r) = 1$)*
Suppose that $V_0(a)$ and operator $\mathbb{T}$ satisfy the conditions of Proposition 1. Suppose also that $\beta(1+r) = 1$. Then all the value functions from the sequence $\{V_t(a)\}_{t=1}^T$ have a common tangent line, which is also tangent to $V_\infty(a)$ at a^* if $a^* > \underline{a}$.
In addition, if $\max\{V_\infty(a), V_0(a)\}$ is not globally concave, the sequence $\{a_t^*\}_{t=2}^T$ of the cutoff levels is infinite (i.e. $T = +\infty$) with $\lim_{t\to+\infty} a_t^* = a^*$.

The proof of Proposition 2 is in the Appendix. Figure 6 illustrates the shape of the agent's

---

$V_\infty(\overline{a}_1) = V_0(\overline{a}_1)$ and $V_\infty'(\overline{a}_1) = V_0'(\overline{a}_1)$, which guarantees that $a^* = a_1^*$ (i.e. the transitory interval is empty) and $V(a) = \max\{V_0(a), V_\infty(a)\}$ is concave.

[24] This argument relies on the assumption that the asset levels $\underline{a}_t$ are interior for all $t = \overline{1,T}$. It is easy to verify that this is indeed true since $\underline{a}_t = \underline{a}$ implies that $V_k(a) < \max\{V_0, .., V_t(a)\}$ for all $k \geq t+1$.

[25] To see this, suppose that $\underline{a_{\infty 0}}$ and $\overline{a_{\infty 0}}$ are the asset levels at which $V_\infty(a)$ and $V_0(a)$ are tangent with their common tangent lines, and $\underline{a_{\infty 1}}$ and $\overline{a_{\infty 1}}$ are the corresponding asset levels for $V_\infty(a)$ and $V_1(a)$. Statement (iii) of Lemma 1, first order and envelope conditions, and $\beta(1+r) = 1$ imply that $V_1'(\overline{a_{\infty 1}}) = V_0'(\overline{a_{\infty 0}})$ and $V_\infty'(\underline{a_{\infty 0}}) = V_\infty'(\underline{a_{\infty 1}})$. The latter implies that $\underline{a_{\infty 0}} = \underline{a_{\infty 1}}$, and thus the line tangent to $V_\infty(a)$ at $\underline{a_{\infty 0}} = \underline{a_{\infty 1}}$ is also tangent to $V_0(a)$ at $\overline{a_{\infty 0}}$ and to $V_1(a)$ at $\overline{a_{\infty 1}}$.

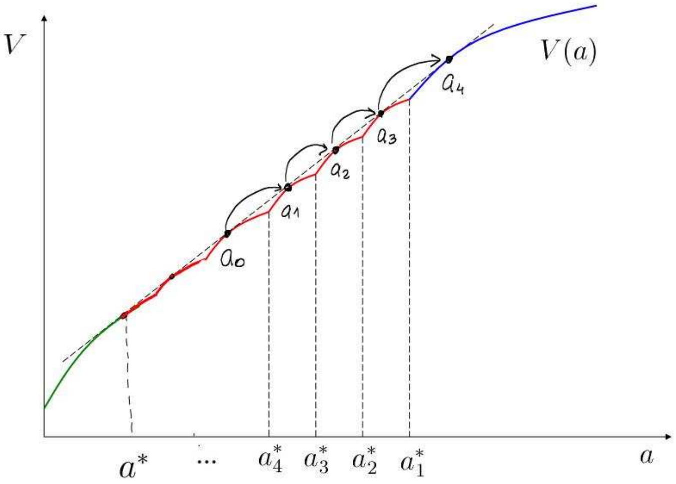

Figure 6: The properties of $V(a)$ and $a'(a)$ if $\beta(1+r)=1$ and (i) of Proposition 1 applies.

value function and the properties of his wealth dynamics inside the transitory interval when $\beta(1+r)=1$. Two observations emerge right away. First, if the consumer's initial wealth a_0 coincides with one of the tangent points inside (a^*, a_1^*), he does not have a gambling motive at any period in his life. Second, $V_0(a)$ appears to be almost linear inside (a^*, a_1^*), implying that the agents with wealth inside this interval are risk neutral with respect to mean-preserving wealth shocks of certain types.[26] In contrast, if fixed adjustment cost η was absent, the optimal housing choice would vary with wealth continuously, the agent's value function would be concave and hence the agent would be risk averse with respect to any wealth (or income) shocks. These observations are in sharp contrast with the predictions of the restricted model discussed in section 3, where discrete housing adjustments create gambling motive and magnify the risk aversion of all the consumers remaining in their initial house.

Note, however, that due to time discreteness, small kinks occur at each cutoff level a_t^*, implying that (a) if the consumer's initial wealth does not coincide with any of the tangent points, he would still like to take a small wealth lottery to eliminate the local non-concavity[27], and (b) if the consumer's initial wealth does no coincide with any of the kink points, he would be risk averse with respect to very small wealth risks[28]. However, the arguments presented

---

[26] For example, if a_0 corresponds to one of the tangent points then in any of the first N periods of his life the consumer would not mind taking a fair lottery randomizing over any combination of the wealth levels $\{a^*, a_0, a_1, ..., a_N\}$.

[27] Note that the welfare benefits of such lottery are likely to be small.

[28] Note, however, that such consumer would not necessarily be more risk averse than the consumer with the same wealth level in the model without housing adjustment costs. The reason is that varying a inside

above do not depend on the length of the time period. Thus it is possible to shorten the length of the time period, thereby allowing consumers to make wealth adjustments more frequently (doing this would only make the model more realistic).[29] Under such modification, the intervals within which $V(a)$ is strictly concave would shrink. A limiting argument can then be used to show that as the length of the time period converges to zero (i.e. the model becomes a continuous time model), the small kinks disappear and the consumer's value function becomes linear inside the interval (a^*, a_1^*), while the boundaries a^* and a_1^* of this interval coincide with the wealth levels at which $V_\infty(a)$ and $V_0(a)$ meet their common tangent line.

To sum up, under $\beta(1+r) = 1$ the possibility of choosing *when* to implement the discrete adjustment eliminates the incentives to take wealth lotteries and creates transitory wealth intervals, within which the agent's risk aversion is reduced relative to the environment where all the goods can be adjusted flexibly.

5.2 Upward transition, $\beta(1+r) \neq 1$

Since Proposition 1 holds for any value of $\beta(1+r) > 0$, it is possible to analyze how risk attitudes of the consumers planning to switch to option $V_0(a)$ (such that $V_0(a) > V_\infty(a)$ for sufficiently large a) would change if $\beta(1+r) \neq 1$. Among other things, such analysis may help determine whether, in the presence of consumption commitments, there exists any relationship between consumers' patience and attitudes towards risk.[30]

Figures 7 and 8 illustrate typical shapes of $V(a)$ for $\beta(1+r) > 1$ and $\beta(1+r) < 1$ respectively. Observe that (9) and (10) hold for any $\beta(1+r)$. Correspondingly, $\beta(1+r) > 1$ would result in $\underline{a}_1 > \overline{a}_2$, while $\beta(1+r) < 1$ would lead to $\underline{a}_1 < \overline{a}_2$. This implies that if the length of the time period is infinitesimally small, those consumers who save in order to switch to $V_0(a)$ in the future become risk-averse when $\beta(1+r) > 1$ and risk-lovers when $\beta(1+r) < 1$. Obviously, the gambling motive, which is present in the restricted model from section 3, disappears in the former case (when consumers are sufficiently patient) but persists

---

each interval (a_t^*, a_{t+1}^*) is equivalent to varying a at the moment of the discrete adjustment, after η is paid and the consumer flexibly chooses the new housing level.

[29] If the time period is made shorter, the model's parameters should be adjusted correspondingly. For example, if each period is split into n equal sub-periods, the following adjustments should be made: $\widehat{\beta} = \beta^{1/n}$, $\widehat{r} = (1+r)^{1/n} - 1$ and $\widehat{y} = \frac{1+r}{r}\frac{\widehat{r}}{1+\widehat{r}}y$.

[30] Note that such relationship is typically absent in the models without consumption commitments. For example, if $\eta = 0$ (so that housing adjusts with wealth continuously), and $y = 0$ and $u(c,h) = v(c) + \alpha v(h)$ with $v(\cdot)$ having constant relative risk aversion (so that the model has a closed form solution), the curvature of the resulting value function is determined solely by the parameters of $v(\cdot)$ and depends neither on β nor on r.

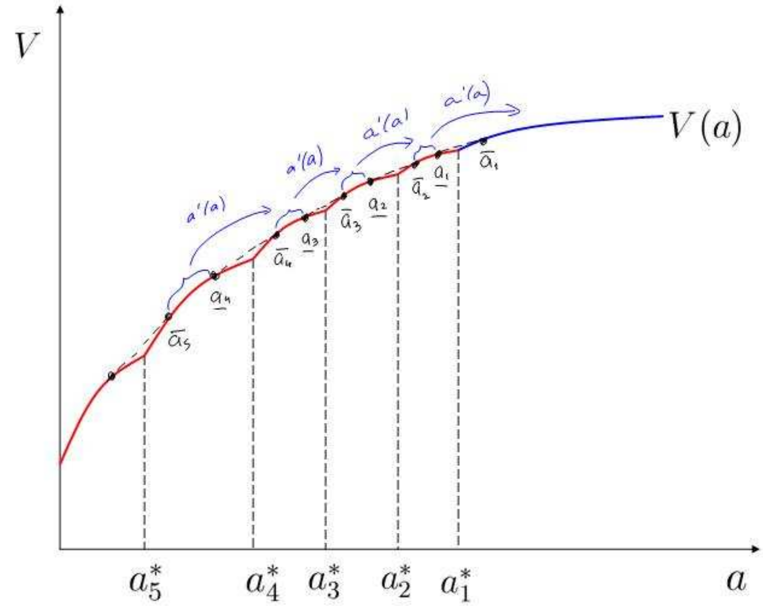

Figure 7: The properties of $V(a)$ and $a'(a)$ if $\beta(1+r) > 1$ and (i) of Proposition 1 applies.

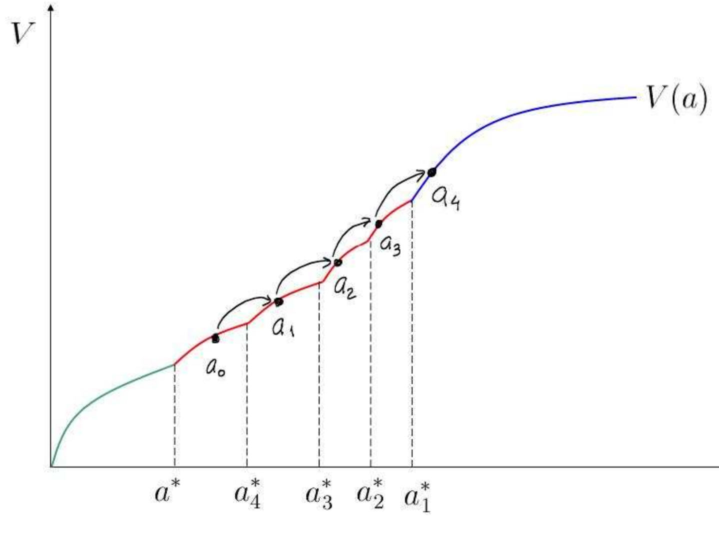

Figure 8: The properties of $V(a)$ and $a'(a)$ if $\beta(1+r) < 1$ and (i) of Proposition 1 applies.

in the latter (when consumers are sufficiently impatient). Note also that even though $V(a)$ is concave within (a^*, a_1^*) when $\beta(1+r) > 1$, the consumers adopting transitory behavior might still be less risk averse than in the environment without consumption commitments (in the sense that a lower premium would be required to make them willing to take an unexpected wealth lottery).

Observe that the above discussion about the relationship between $\beta(1+r)$ and transiting consumer's risk attitudes is applicable in any dynamic discrete choice model with perfect foresight, in which conditions (a) and (b) of Proposition 1 can be verified. Recall, however, that the housing model introduced in section 2 and characterized in more detail in section 6 assumes that $\beta(1+r) = 1$. This assumption simplifies the analysis in two dimensions: first, it guarantees that housing adjustments occur only once; second, it allows to verify analytically that conditions (a) and (b) of Proposition 1 hold (as is done later in Proposition 3). If $\beta(1+r)$ were different from 1, repeated housing adjustments would occur, and verifying conditions (a) and (b) would be substantially more difficult (since $V_0(a)$ would correspond to the value of making the next – not the only – discrete adjustment, and it would depend on what kind of housing adjustments and how frequently are made thereafter). However, it is still possible that even with $\beta(1+r) \neq 1$ conditions (a) and (b) (for the appropriately defined $V_0(a)$) are satisfied,[31] in which case the above discussion concerning the relationship between the value of $\beta(1+r)$ and transiting consumer's risk attitudes would also be relevant in the particular the housing model.

5.3 The role of $\beta(1+r)$ in determining transitory consumers' risk attitudes: intuition

At first glance, it is somewhat surprising that risk attitudes (risk averse/risk loving/risk neutral) of the consumers adopting transitory behavior are solely determined by the value of $\beta(1+r)$ and do not depend on the curvature of the instantaneous utility function $u(\cdot, h)$ or of the value function $V_0(a)$. To understand the driving force behind this result, I construct below two examples suggesting why $\beta(1+r)$ plays such an important role and then explain how the lessons learnt from these particular examples can be applied to dynamic discrete choice models with perfect foresight.

Example 1: Wealth accumulation and utility gains

---

[31] In fact, I have performed a number of numerical exercises with $\beta(1+r) \neq 1$ and in all of them $V_0(a)$ indeed satisfied (a) and (b) of Proposition 1.

Consider the agent who saves $s > 0$ in period $t = 0$ and by the time when the total cumulative return on his savings becomes equal to Y, the agent can take an action which would raise his life-time utility by Δ (measured in utils). Suppose that in period 0 the agent is allowed to either save s in a risk-free bond or to take a fair wealth lottery (with the expected payoff of s) and then save its outcome in a risk-free bond. If the lottery is successful, the agent would be able to experience the utility jump Δ sooner; if the outcome of the lottery is not successful, the utility jump would occur later. In order to figure out whether the agent would be willing to take the lottery first, let us compute the present value $PV(s)$ of the utility gain associated with the future utility jump Δ if the agent saves s in a risk-free bond.

For brevity, assume (for this part only) that time is continuous. The agent can accumulate amount Y by time period T such that $s \exp(rT) = Y$. The present value of the utility jump is equal to $PV(s) = \exp(-\rho T)\Delta$ (where $\beta = \frac{1}{1+\rho}$), which could be rearranged as

$$PV(s) = \exp(-\rho T)\Delta = (\exp(-rT))^{\rho/r}\Delta = (s/Y)^{\rho/r}\Delta.$$

Notice that $PV(s)$ is strictly concave in s if $\rho < r$ (which in discrete time is equivalent to $\beta(1+r) > 1$), strictly convex if $\rho > r$ (i.e. $\beta(1+r) < 1$) and is linear if $\rho = r$ (i.e. $\beta(1+r) = 1$). Thus the consumer would prefer to invest in a risk-free bond in the first case, to gamble his initial wealth s in the second, and would be indifferent between the two options in the third case. Recall that in exactly the same conditions on $\beta(1+r)$ determine whether the dynamic discrete choice model studied in the paper the agent exhibiting transitory behavior is risk averse, risk lover or risk neutral.

The predictions in this example are driven by the interaction between two forces. First, exponential discounting implies that the agents are risk lovers with respect to the timing of the utility gain (since its present value $\exp(-\rho T)\Delta$ is convex in T). Second, exponential return on savings implies that the mean-preserving lottery over initial wealth increases the expected waiting time T till the utility gain occurs (since $T = \frac{\ln Y - \ln s}{r}$ is convex in T). While the first force creates incentives for risk taking, the second acts against it. When the consumers are sufficiently impatient, the former effects dominates because the degree of convexity of $\exp(-\rho T)\Delta$ rises, and the consumers become risk lovers.

Example 2: Separable $u(c, h)$, no borrowing constraints

Next, I illustrate that the previous example can be mimicked by a special case of the model set up in section 2. Suppose that the utility function is separable in consumption and housing, $u(c,h) = v_c(c) + v_h(h)$, and that there are no borrowing constraints. Then, for any possible new optimal optimal housing level h^*, the agent solves the following decision problem:[32]

$$\begin{aligned} &\max_{\{c_t\},T} \sum_{t=0}^{+\infty} \beta^t v_c(c_t) + \frac{v_h(h_0)}{1-\beta} + \beta^T \frac{v_h(h^*) - v_h(h_0)}{1-\beta} \\ &\text{s.t.} \quad \frac{1+r}{r} y + a_0 = \sum_{t=0}^{+\infty} \frac{c_t}{(1+r)^t} + \frac{1+r}{r} h_0 + + \frac{1}{(1+r)^T} [\eta + \frac{1+r}{r} (h^* - h_0)]. \end{aligned} \tag{11}$$

The similarities between decision problem (11) and Example 1 studied above are obvious and can be interpreted in the following way. In period 0 the agent borrows against his future income and opens two risk-free bank accounts: the savings in the first bank account are used to finance the stream of consumption expenses $\{c_t\}$ and housing payments of size h_0 throughout lifetime; while the savings in the second bank account are used to finance the switch from h_0 to h^* and additional housing payments of size $h^* - h_0$ after the switch. The switch will occur once the agent accumulates $Y = \eta + \frac{1+r}{r}(h^* - h_0)$ in the second bank account, at which time the agent's life-time utility will increase by $\Delta = \frac{v_h(h^*) - v_h(h_0)}{1-\beta}$. Thus, in accordance with findings of Example 1, the agent should strictly prefer to invest his savings in the second bank account in a safe asset if $\beta(1+r) > 1$, take a gamble and save the lottery's payoff in a safe asset if $\beta(1+r) < 1$, and would be indifferent between the two options if $\beta(1+r) = 1$.

General intuition

The two above examples suggest that the risk attitudes of the consumers inside the intermediate interval (a^*, a_T^*) are explained by the fact that the savings decision of the consumer who eventually plans to switch to a new option could be separated into two parts. More specifically, this agent makes savings for two different purposes: (i) to smooth the marginal utility of his flexible consumption $u_1(c, h_t)$ (which is the same as the marginal value $V'(a)$) over time and (ii) to raise his life-time utility level at the moment when the switch happens. The second type of savings governs the agent's risk preferences during the path of wealth accumulation towards the switch; that is why the findings from Example 1 also apply in a general model.

The above observations uncover an interesting (and, to my knowledge, novel) relationship between patience and attitudes towards risk arising in dynamic discrete choice models: while

---

[32] The decision problem (1) can be split into two steps: solve (11) for any possible h^*, and then choose h^* that delivers the highest value.

saving in order to switch to a more attractive option *less patient* agents also tend to be *more risk tolerant.* All the results presented up to now can be applied in a wide variety of dynamic discrete choice models, for which conditions of Proposition 1 can be verified. For instance, if applied to an occupational choice model with borrowing constraints, it suggests that less patient workers should be willing to take more risky jobs (or make more risky investments) if they are planning to become entrepreneurs in the future. Similarly, among consumers saving for their (endogenously timed) retirement, those with lower discount factor (e.g. due to lower survival probability) should be more willing to take risk actions. Alternatively, these findings can also be used in costly technology adoption models to argue that, while saving in order to switch to a more productive technology, the firms (or countries) with lower discount factor should be more willing to undertake risky projects. It would be interesting to see whether these predictions find support in the data and, if they do, to study their further implications.

5.4 Downward transitions

All the steps of the above analysis can be immediately applied to characterize risk attitudes of the consumers *transiting downwards* (i.e. dissaving while planning to switch to a new option in the future). Not surprisingly, the results are symmetric: such consumers are risk averse along the transition path if $\beta(1+r) < 1$, risk neutral if $\beta(1+r) = 1$ and are risk lovers if $\beta(1+r) > 1$. Similarly, Proposition 2 also applies in this case: by optimally choosing the moment of switching to a smaller house and the path of wealth during the transition, these consumers are able to eliminate the kink in the value function induced by the discreteness of housing choice if $\beta(1+r) = 1$.[33]

Comparison of the predictions for upward and downward transiting consumers indicates that risk loving behavior arises if during the transition toward a new option the agent's wealth adjusts in the opposite direction from the one that would be induced by the value of $\beta(1+r)$ if there were no fixed adjustment costs. For instance, when $\beta(1+r) < 1$ and $\eta = 0$, the agent would choose to gradually consume his wealth out while gradually decreasing his consumption of both goods; but if $\eta > 0$ induces the agent to start accumulating assets in order to finance the switch to a new option in the future, gambling is beneficial because it either allows him to shorten (or completely eliminate) the period of such 'undesirable' saving (if the lottery's outcome is successful) or makes the option that requires extra savings unattractive (if the lottery's outcome is unsuccessful).

---

[33] Thus the statement of Proposition 2 does not explicitly require that $V_0(a) > V_\infty(a)$ for large a.

6 The solution to the housing model

The recursive solution approach developed in the previous section (as well as the resulting implications regarding risk attitudes of the transiting consumers) can be applied to the housing model set up in section 2 only if $V_0(a)$ and $V_\infty(a)$ defined in (5) and (8) respectively satisfy conditions (a) and (b) of Proposition 1. However, intuition suggests that if no restrictions are imposed on the set from which new housing level h^* can be chosen, condition (b) of Proposition 1 should be violated: if the agents' current wealth is substantially different from the one at which the initial housing allocation h_0 is optimal, they would want to make a housing adjustment immediately (as opposed to keeping initial h_0 forever); the relatively poor agents would move into a smaller house, and the relatively rich ones would choose a bigger house. Hence, $V_0(a)$ defined in (5) is likely to have at least two intersections with $V_\infty(a)$, with $V_0(a) > V_\infty(a)$ for sufficiently small and sufficiently large a. Thus Proposition 1 cannot be directly applied to solve (1).

However, the agent's decision can be split into two steps: first, (1) can be solved under two separate restrictions – one that the agent can only move into a bigger house (i.e. h^* can only be chosen from $\Gamma^+(h_0) = \{h : h \geq h_0\}$) and the opposite, saying that the agent can only move into a smaller house (i.e. h^* can be only chosen from $\Gamma^-(h_0) = \{h : h \leq h_0\}$); and after that the agent's choice between these two options can be characterized. Denote by $V_0(a; h^* \geq h_0)$ and $V_0(a; h^* \leq h_0)$ the value functions solving (5) under the restrictions $h^* \in \Gamma^+(h_0)$ and $h^* \in \Gamma^-(h_0)$ respectively. Obviously, $V_0(a; h^* \geq h_0) > V_\infty(a)$ for large a and $V_0(a; h^* \geq h_0) < V_\infty(a)$ for small a. In fact, for sufficiently small a, $V_0(a; h^* \geq h_0) = V_\infty(a-\eta)$ when the restriction $h^* \geq h_0$ is imposed in (5).[34] Similarly, $V_0(a; h^* \leq h_0) > V_\infty(a)$ for small a and $V_0(a; h^* \leq h_0) = V_\infty(a-\eta) < V_\infty(a)$ for large a. It is possible to show that, under some additional conditions (namely, weak supermodularity of $u(c,h)$ and no borrowing limits), both $V_0(a; h^* \geq h_0)$ and $V_0(a; h^* \leq h_0)$ satisfy the conditions of Proposition 1.[35] Hence, in accordance with the findings in the previous section, the choice between staying in h_0 forever and switching to a bigger house generates a unique upward transitory interval $(a^*_H, a^*_{1,H}]$, while the choice between staying in h_0 forever and switching to a smaller house leads to the occurrence a unique downward transitory interval $[a^*_{1,L}, a^*_L)$.

---

[34] This is because at small a the agents choose the lowest feasible h^*, i.e. $h^* = h_0$.

[35] Chetty and Szeidl (2007) show that $V_0(a; h^* \geq h_0)$ and $V_0(a; h^* \leq h_0)$ each have unique intersection with $V_\infty(a)$ under the assumption that $u(c,h)$ is separable, $u(c,h) = v_1(c) + v_2(h)$, and $v_1(c)$ and $v_2(c)$ are CRRA. Proposition 2 below establishes the same property under weaker conditions (namely, only supermodularity of $u(c,h)$), and it also verifies that $V_0(a)$ and $\mathbb{T}V_0(a)$ satisfy single-crossing, which is needed for the characterization of transitory intervals.

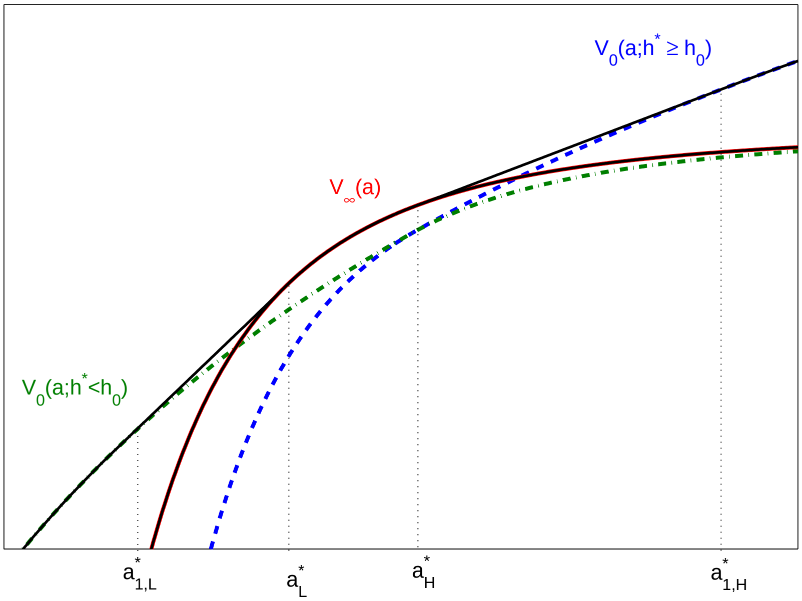

Figure 9: The optimal value V(a), $\beta(1+r) = 1$ and no borrowing constraints

Figure 9 illustrates $V_\infty(a)$ (solid black curve), $V_0(a;h^* \geq h_0)$ (dash-and-dot green curve) and $V_0(a;h^* \leq h_0)$ (dashed blue curve). Recall that in the previous section (Proposition 2) it was shown that under $\beta(1+r) = 1$ the cutoffs a^*_L and a^*_H correspond to the asset levels at which $V_\infty(a)$ is tangent to the common tangent lines of $V_\infty(a)$ and $V_0(a;h^* \leq h_0)$, and of $V_\infty(a)$ and $V_0(a;h^* \geq h_0)$, respectively. It is also not surprising that $a^*_L < a^*_H$ holds, i.e. the transitory intervals are disjoint and there arises a unique *inaction* region $[a^*_L, a^*_H]$, in which the agent's optimal housing choice remains at initial h_0.[36] Hence, the agent's housing decision is fully characterized by the four cutoff levels $a^*_{1,L} < a^*_L < a^*_H < a^*_{1,H}$. This is formally stated in the Proposition below and is proven in the Appendix.

PROPOSITION 3 *(characterization of the optimal housing choice in (1))*
Suppose that $\beta(1+r) = 1$, $u(c,h)$ is supermodular (i.e. $u_{12}(c,h) \geq 0$) and there are no borrowing constraints (i.e. $\underline{a} = -\infty$). Then the solution to (1) is characterized by four cutoff wealth levels $a^*_{1,L} < a^*_L < a^*_H < a^*_{1,H}$:

- *if $a_0 \in [\underline{a}, a^*_{1,L}]$ then the agent chooses to switch immediately to some $h^*(a_0) < h_0$;*

---

[36] Formally, it follows from the fact that $\max\{V_0(a;h^* \geq h_0), V_0(a;h^* \leq h_0)\}$ is concave – since it corresponds to the value of switching to *any* optimal h^* immediately – and that the set of a for which $V_\infty(a) > \max\{V_0(a;h^* \geq h_0), V_0(a;h^* \leq h_0)\}$ is non-empty – since $\min\{V_0(a;h^* \geq h_0), V_0(a;h^* \leq h_0)\} = V_\infty(a - \eta) < V_\infty(a)$.

- *if* $a_0 \in (a^*_{1,L}, a^*_L)$ *then the agent chooses a declining over time wealth profile and makes housing adjustment in* $T(a_0)$ *periods, where* $T(a_0)$ *is increasing in* a_0*;*
- *if* $a_0 \in [a^*_L, a^*_H]$ *then the agent remains in* h_0 *forever;*
- *if* $a_0 \in (a^*_H, a^*_{1,H})$ *then the agent chooses an increasing over time wealth profile and makes housing adjustment in* $T(a_0)$ *periods, where* $T(a_0)$ *is decreasing in* a_0*;*
- *if* $a_0 \in [a^*_{1,H}, +\infty)$ *then the agent switches to some* $h^*(a_0) > h_0$ *immediately.*

In Proposition 3, weak supermodularity of $u(c, h)$ and the assumption of borrowing limits are needed to verify conditions (a) and (b) of Proposition 1. Weak supermodularity is a commonly used assumption, which also holds if $u(c, h)$ is separable in c and h. The absence of borrowing constraints is a more restrictive feature. Note, however, that it is only needed to verify the uniqueness of the downward transitory interval. The intuitive explanation is very straightforward: In the absence of borrowing constraints, poor consumers would prefer to move into a smaller house because this frees up some life-time wealth for food consumption. However, when a borrowing limit is imposed and the consumer's current wealth is close to the borrowing limit, moving to a new house (if feasible) cannot be optimal if the adjustment cost is relatively high (close to or exceeds y) because such decision would considerably lower food consumption in the current period (or even make positive consumption levels not feasible). If this happens, $V_0(\underline{a}; h^* \leq h_0) < V_\infty(\underline{a})$ and, correspondingly, $V_0(a; h^* \leq h_0)$ and $V_\infty(a)$ might cross more than once. Notice that such an issue does not appear when the consumers are allowed to move only into a bigger house because $V_0(a; h^* \geq h_0) < V_\infty(a)$ for small a anyway. Thus the assumption of no borrowing constraints in Proposition 3 is relevant only for establishing the downward transitory interval, but it actually is not required in the characterization of the upward transitions.

As illustrated on Figure 9, under $\beta(1+r) = 1$ the optimal $V(a)$ effectively coincides with the concave envelope of $\max\{V_\infty(a), V_0(a; h^* \leq h_0), V_0(a; h^* \geq h_0)\}$, with the exception that, due to time discreteness, very small kinks occur within the transitory intervals.[37] Had the option of postponing housing adjustment been eliminated, as in the example considered in section 3, the agent's value would be given by $\max\{V_\infty(a), V_0(a; h^* \leq h_0), V_0(a; h^* \geq h_0)\}$. As can be seen, the possibility of choosing *when* to make a housing adjustment alters the shape of the value function in two ways: first, it eliminates the gambling motive, and, second,

---

[37] Note that such kinks are not even visible on Figure 9, even though it plots $V(a)$ that was obtained in one of the numerical exercises described in the following section.

it reduces the size of the *inaction* interval within which the presence of the adjustment cost magnifies consumers' risk aversion, and instead creates *transitory* intervals within which the consumers are risk neutral (i.e. less risk averse than in the absence of consumption commitments). The latter observation has important implication for the possible effects of consumption commitments on consumers' risk preferences. A number of existing studies (dating back to Grossman and Laroque (1990)) have suggested that, since consumption commitments magnify risk aversion of some consumers by leading to appearance of inaction intervals, they might be useful in explaining the equity premium puzzle. This paper, however, points out, that along with magnifying risk aversion with respect to small risk for some consumers, consumption commitments also reduce the risk aversion of the others, which means that, at least theoretically, the overall effect of consumption commitments on aggregate risk preferences, is ambiguous. To address this ambiguity, one has to analyze whether (and under which conditions) a number of consumer adopting transitory behavior may be significant in comparison with the number of consumers in the inaction region. In the next section I develop a series of quantitative exercises designed to address this question and find that the presence of transitory intervals may indeed have important quantitative implications.

7 Numerical Analysis

In this section I develop a series of numerical exercises in order to analyze whether the transitory behavior – the main focus of this paper – is likely to arise in the dynamic model introduced in section 2. In general, consumers might decide to switch to a new house (immediately or with a delay) if their initial housing choice ends up being, for some reason, suboptimal. This might happen, for instance, if an income shock arrives after the initial housing commitment is made.

Intuitively, two separate features of the model might independently create incentives for transitory behavior: positive interest rate $r > 0$ and the presence of the borrowing constraints. When $r > 0$, the consumer can decrease the present value of the adjustment cost (and increase the present value of his life-time wealth) by postponing housing adjustment for a while. When borrowing is restricted, it might be too costly (or even not feasible) to pay the adjustment cost η in period 0 and the consumer might choose to wait for a while till he accumulates sufficient resources to finance transition. As it was mentioned earlier, these features are absent from the model in Chetty and Szeidl (2007), therefore transitory behavior never appears in their environment. In what follows I attempt to identify how important each of these factors might be for stimulating transiting behavior.[38]

The parameter values are chosen to be the same as in the benchmark simulation of Chetty and Szeidl (2007) (see column 2 of Table 2). The instantaneous utility function is separable in consumption and housing:

$$u(c,h) = \frac{c^{1-\sigma_c}}{1-\sigma_c} + \mu \frac{h^{1-\sigma_h}}{1-\sigma_h},$$

where $\sigma_c = 4$ and $\sigma_h = 1$. A period is set to be a year, so $r = 0.0417$ and $\beta = \frac{1}{1+r} = 0.96$ and .[39] Per period income is normalized to $y = 1$, and $\mu = 8$ implies that the agent foreseeing constant income for the rest of his life chooses to spend 50% of it on housing. In this case the initial housing commitment for the agents with $a_0 = 0$ is equal to $h_0 = 0.5$. The adjustment cost η is equal to 10% of the total life-time value of the initial housing commitment and I will perform a comparative statics exercise with respect to this parameter.

7.1 Transitions induced by $r > 0$, myopic consumers

To emphasize the role of $r > 0$ for generating transitory behavior, I first consider the model without borrowing constraints. The value functions obtained under the chosen parameterizations were shown on Figure 9. Figure 10 illustrates that there are substantial differences in risk attitudes between the consumers with different wealth level. It plots the percentage risk premium that would be required to make consumers willing to gamble their current period income in a 50/50 lottery in two different models, with and without consumption commitments. If there are no adjustment costs, such risk premium gradually declines with wealth (since $u(c,h)$ has DARA property). When the fixed adjustment cost of size η is imposed, the consumers with the wealth level around zero (where $h_0 = 0.5$ is optimal) become significantly more risk averse than in the model without adjustment costs. At the same time, consistently with the theoretical findings, the consumers in the transitory wealth intervals are risk neutral – they would be willing to take this income lottery even if it does not offer any premium.

Judging from Figures 9 and 10, the transitory intervals appear to be wide in comparison with the the inaction interval. In the benchmark model, an increase in the initial wealth in

---

[38] There is a short section in Chetty and Szeidl (2007), which argues that borrowing constraints magnify local risk aversion with respect to a particular type of negative income shocks. I complement their discussion by illustrating that borrowing limits can lower risk aversion with respect to positive income shocks.

[39] Recall that this is different from parameterization in Chetty and Szeidl (2007), where $\beta = 1$ and $r = 0$.

Figure 10: Comparison of the risk premia (Δ) needed to make the consumer willing to gamble their current period income in the models with and without adjustment costs (the lottery is a 50/50 gamble between 0 and $2(y+\Delta)$).

the range from 1.57 to 8.33 (157% to 833% of annual income) after the initial housing choice is made would induce the consumer's transition toward a bigger house, and a decline in the initial wealth of the size of 182% to 640% of annual income would give impetus to a downward transition. When there are no borrowing constraints, changes in the initial wealth could be replicated by permanent changes in the annual income (since only the total life time wealth is important for consumer's decision). Thus, in the benchmark model, the consumer would start saving for a bigger house if in period zero, after the initial commitment of $h_0 = 0.5$ is made, he receives an unexpected permanent shock to his income in the range from 6% to 34% (i.e. the new income level varies between 1.06 and 1.34). Similarly, a downward transition would start if per period income drops unexpectedly by 7% to 22%.[40]

Table 1 reports how the width of the transitory interval adjusts when η falls. As can be seen, even when the adjustment cost is fairly small, the ranges of income shocks which would put the consumer into a transiting interval (and make him locally risk neutral) are significant in comparison with the range of income shocks for which the agent would remain inside the inaction interval (and be more risk averse than in the model without consumption commitments).

---

[40] Income shocks of this size are not uncommon in the data and most numerical macroeconomic models with incomplete markets assume that the standard deviation of the annual income is around 20% (e.g. Hugget 1996).

Table 1: Income shocks generating transiting behavior
($r > 0$, myopic behavior, no borrowing constraints)

η	% of $\frac{1+r}{r}h_0$	negative y shock	positive y shock
1.250	0.100	(-0.22,-0.07)	(0.06,0.34)
0.625	0.050	(-0.15,-0.05)	(0.05,0.23)
0.312	0.025	(-0.12,-0.03)	(0.04,0.15)

Table 2: Distribution of *ex post* housing decisions
(rational consumers, $r > 0$, no borrowing constraints)

σ	h_0 at $a_0 = 0$	$h^* < h_0$ at $T = 0$	$h^* < h_0$ at $T > 0$	h_0 forever	$h^* > h_0$ at $T > 0$	$h^* > h_0$ at $T = 0$
0.20	0.45	0.09	**0.12**	0.38	**0.30**	0.11
0.13	0.46	0.02	**0.09**	0.54	**0.32**	0.03
0.09	0.47	0.01	**0.04**	0.67	**0.27**	0.01

Table 3: Positive income shocks generating transiting behavior
(introducing borrowing constraints)

η	% of $\frac{1+r}{r}h^1$	no constraints	$a_t \geq 0$ is imposed
1.250	0.100	(0.06,0.34)	(0.06,0.81)
0.625	0.050	(0.05,0.23)	(0.05,0.46)
0.312	0.025	(0.04,0.15)	(0.04,0.23)

7.2 Transitions induced by $r > 0$, rational consumers

The previous numerical example summarized in Table 1 describes the behavior of myopic consumers, whose initial housing decision is made under the assumption that there would be no income shocks after h_0 is chosen. If consumers foresee the possibility of receiving a permanent income shock after the commitment is made, their initial housing choice would be different, and, correspondingly, the transitory intervals would adjust. Thus in the next exercise I model rational consumers, who make their initial housing choice knowing the distribution of income shocks that would arrive afterwards.

Assume that *ex post* annual income is drawn from a normal distribution with mean 1 and variance σ^2. After the shock's initial realization in period 0, the level of income remains constant forever. Table 2 lists the values of initial housing commitments made by rational consumers in period 0 for different levels of σ as well as the distribution across the types of housing adjustments made after the income shock is realized. For instance, when the standard deviation of the income shock is equal to 0.2, the initial housing choice falls to $h_0 = 0.45$ (the consumers self-insure against low income realizations). After the income shocks arrive, 9% of the consumers move to a smaller house immediately, 38% remain in their initial house h_0, and 11% decide to move into a bigger house right away. The remaining 42% of the agents start adjusting their wealth in order to eventually move into a new house. Out of them, 12% are transiting downwards and 30% are transiting upwards. Not surprisingly, the smaller is the income uncertainty, the bigger is the fraction of agents who choose to remain in their initial house forever. However, even though the fraction of consumers switching right away becomes almost negligible as σ falls, the amount of transiting consumers remains significant. These observations suggest that the transiting behavior occurring due to $r > 0$ might have a significant quantitative impact on the aggregate attitudes toward risk.

7.3 Transitions induced by the borrowing constraints

Next, I analyze to what extent borrowing constraints can contribute to the appearance of transiting behavior. Since in the presence of the borrowing constraints downward transitions are hard to characterize analytically, (as conditions (a) and (b) of Proposition 2 are satisfied only if h^* is chosen from $\{h : h \geq h_0\}$), I focus in this exercise on upward transitions only. Table 3 compares the ranges of positive income shocks that induce transiting behavior of myopic consumers (the ones who commit to $h_0 = 0.5$ initially) in the models with and without borrowing constraints. As can be seen, imposing the restriction $a_t \geq 0$ considerably broadens the range of permanent income shocks, after the realization of which consumers become risk neutral.

Finally, it is worth pointing out that this paper describes a situation, in which the presence of the borrowing constraints reduces the welfare cost of risk for a subset of consumers. This is in sharp contrast with the traditional findings in incomplete markets models (e.g. Aiyagari (1993)), where borrowing constraints make agents more risk averse because consumption of constrained consumers must drop if they get hit by a negative shock. This paper illustrates that borrowing limits, in conjunction with fixed adjustment costs, may generate transitory intervals, within which agents become indifferent to risk (or even like it if $\beta(1+r) < 1$ and they are transiting upwards).

To sum up, a series of simple quantitative exercises presented in this section suggests that the occurrence of transitory behavior in the models with fixed adjustment costs (or other features leading to lumpy adjustments) might play an important role in shaping the economy-wide demand for risky assets. At the same time, I must acknowledge that the model studied in the paper is quite stylized and thus the quantitative results summarized should be primarily considered as a motivation for a more careful quantitative analysis. To further evaluate the impact of fixed adjustment costs on aggregate preferences for risk, a more realistic model, and a more advanced numerical analysis are required. One possible direction would be to analyze a Bewley-type model with uninsured idiosyncratic income risk and borrowing constraints, which is extended to include two goods (consumption and housing), one of which (housing) is subject to fixed adjustment costs. A model of this type would generate endogenous distribution of consumer's wealth and current housing levels. Together with the current income realizations, these two state variables would determine whether a particular consumer is in the process of transiting towards a new house, which in turn would affect the consumer's risk attitudes. Thus such a model would generate endogenous distribution of risk attitudes in the population. By comparing such a model with its equivalent without fixed adjustment costs, one can evaluate how the presence of the fixed adjustment costs and their affect on the incidence of transitory behavior affect aggregate attitudes towards risk and aggregate demand for risky assets.

8 Final Remarks

This paper illustrated that some of the predictions regarding the effects of consumption commitments on consumers' attitudes towards risk derived from static models do not necessarily hold, and may even reverse, in a dynamic environment. It showed that (i) consumers can eliminate non-convexities in their value function by delaying housing adjustments and that (ii) the consumers who are planning to switch to a different house in the future are likely to be more risk tolerant than the consumers with the same asset level in the model without fixed adjustment costs. These findings suggest that consumption commitments (i) do not necessarily explain why consumers simultaneously buy lotteries and insurance and (ii) have a weaker potential for explaining the equity premium puzzle than it has been suggested in the previous literature.

In addition to shedding the light on these effects of consumption commitments on risk attitudes, the paper makes three other contributions. First, it illustrates that the risk attitudes of the consumers planning to make an adjustment in the future are directly related to how patient they are: among those who are saving in order to make a discrete adjustment ilater on, the least patient ones also end up being the most risk tolerant. Second, it provides an example of the economic environment in which, contrary to conventional wisdom, borrowing constraints may increase risk tolerance (at least for some consumers) by increasing the incidence of transitory behavior. Third, it makes a methodological contribution to the literature on dynamic discrete choice by weakening the set of sufficient conditions under which a full analytical characterization can be provided.

The model developed in the paper is quite stylized. The simplicity of the modeling environment allows to keep the results as clean as possible and focus solely on the motivating questions. However, it would be interesting to analyze to what extent the results presented in the paper are robust to various modifications that would make the environment more realistic. One question is how uninsured uncertainty in per period income y (as in Beweley-type models) might change the results. Unfortunately, the methodology developed in section 4 does not extend to such modification. However, a numerical analysis would be straightforward to perform. Intuitively, the main results would persist if the the variance in y is sufficiently small or if shocks to y are sufficiently persistent. As discussed in the end of section 7, under such modification, the model would generate an endogenous distribution of wealth and housing allocations, which would translate into an endogenous distribution of risk attitudes and allow to quantify the effects of consumption commitments on aggregate risk attitudes using a more realistic model.

9 References

Aiyagari, R. (1994). Uninsured Idiosyncratic Risk and Aggregate Saving. *The Quarterly Journal of Economics,* Vol. 109(3), pp. 659-684

Albuquerque, R. and H. Hopenhayn. Optimal Lending Contracts and Firm Dynamics. *The Review of Economic Studies*, 2004, 71(2), pp. 285-314.

Attanasio, O. (2000). Consumer Durables and Inertial Behaviour: Estimation and Aggregation of (S, s) Rules for Automobile Purchases. *The Review of Economic Studies*, 67(4), pp. 659-677.

Athreya, K. Welfare Implications of the Bankruptcy Reform Act of 1999. *Journal of Monetary Economics*, 49, pp. 1567-1595.

Bailey, M., Olson, M. and P. Wonnacot (1980). The Marginal Utility of Income Does Not Increase: Borrowing, Lending, and Friedman-Savage Gambles. *American Economic Review*, 70, pp. 372-379.

Bohacek, R. (2006). Financial Constraints and Entrepreneurial Investment. *Journal of Monetary Economics*, 53(8), pp. 2195-2212.

Buera, F. (2006). Persistency of Poverty, Financial Frictions and Entrepreneurship. *Unpublished manuscript.*

Cagetti, M. and M. DeNardi (2006). Entrepreneurship, Frictions and Wealth. *Journal of Political Economy*, 114(5), pp. 835 - 870.

Chetty, R. and A. Szeidl (2007). Consumption Commitments and Risk Preferences. *The Quarterly Journal of Economics*, 122, pp. 831–877.

Dixit, A. and R. Pindyck (1993). *Investment Under Uncertainty.* Princeton University Press.

Eberly, J. (1994). Adjustment of Consumers' Durables Stocks: Evidence from Automobile Purchases. *Journal of Political Economy* CII, pp. 403 - 436.

Flavin, M. and S. Nakagawa (2008). A Model of Housing in the Presence of Adjustment Costs: A Structural Interpretation of Habbit Persistence. *American Economic Review* 98, pp. 474–495.

Fratantoni, M. (2001). Homeownership, Committed Expenditure Risk, and the Stockholding Puzzle. *Oxford Economic Papers* 53, pp. 241–259.

Friedman, M. and L. Savage (1948). The Utility Analysis of Choices Involving Risk. *The Journal of Political Economy*, 56, pp. 279-304.

Fukushima, K. (2005). Asset Pricing Implications of Precommitted Consumption. *Manuscript.*

Greenwood, J. and B. Jovanovic (1990). Financial Development, Growth and the Distribution of Income. *The Journal of Political Economy*, 98, pp. 1076 – 1107.

Hartley, R. and L. Farrel (2002). Can Expected Utility Theory Explain Gambling? *American Economic Review*, 92, pp. 613-624.

Hugget, M. (1996). Wealth Distribution in Life-Cycle Economies. *Journal of Monetary Economics*, 28, pp. 469-494.

Khan, A. and B. Ravikumar. Costly Technology Adoption and Capital Accumulation. *Review of Economic Dynamics*, 5, pp. 489–502.

Lucas, R. (1978). On the Size Distribution of Business Firms. *Bell Journal of Economics*, 9(2), pp. 508-523.

Marshall, D. and N. Parekh (1999). Can Costs of Consumption Adjustment Explain Asset Pricing Puzzles? *The Journal of Finance*, 54, pp. 623-654.

Postlewaite, A., Samuelson, L. and D. Silverman (2006). Consumption Commitments and Employment Contracts. *Unpublished manuscript.*

Quadrini, V. (2000). Entrepreneurship, Saving and Social Mobility. *Review of Economic Dynamics*, 3, pp. 1-40.

Rogerson, R. "Indivisible labor, lotteries and equilibrium." *Journal of Monetary Economics,* 21, pp. 3–16.

Shore, S. and T. Sinai (2006). Commitment, Risk and Consumption: Do Birds of a Feather Have Bigger Nests? *NBER Working Paper 11588.*

Stokey, N. (2009). Moving Costs, Nondurable Consumption, and Portfolio Choice, *Journal of Economic Theory*, 144 (6), pp. 2419–2439.

Vereshchagina, G., and H. Hopenhayn (2009). Risk Taking by Entrepreneurs. *American Economic Review*, 99(5), pp. 1808 - 30.

Warren, E. and A. Tyagi (2003). *The Two-Income Trap.* New York: Basic Books.

10 Appendix

Proof of Lemma 1:

It is convenient to establish (ii) of Lemma 1 first. To verify that for any $\widehat{a}$, for which $\mathbb{T}F(\widehat{a}) \geq \mathbb{T}G(\widehat{a})$, it must be true that $a'_F(\widehat{a}) \geq a^*$, we construct an argument by contradiction. Suppose that $a'_F(\widehat{a}) < a^*$. Then, since $F(a) < G(a)$ for all $a < a^*$, it must be that $F(a'_F(\widehat{a})) < G(a'_F(\widehat{a}))$. At the same time, $a'_F(\widehat{a})$ is a feasible saving policy for the consumer maximizing $\mathbb{T}G(\widehat{a})$. Thus

$$\begin{aligned}\mathbb{T}G(\widehat{a}) &\geq u\left(\widehat{a} + y - h_0 - \frac{a'_F(\widehat{a})}{1+r}, h_0\right) + \beta G(a'_F(\widehat{a})) \\ &> u\left(\widehat{a} + y - h_0 - \frac{a'_F(\widehat{a})}{1+r}, h_0\right) + \beta F(a'_F(\widehat{a})) = \mathbb{T}F(\widehat{a}),\end{aligned}$$

which obviously contradicts to $\mathbb{T}F(\widehat{a}) \geq \mathbb{T}G(\widehat{a})$.

A similar argument is used to verify that for any $\widehat{a}$, for which $\mathbb{T}F(\widehat{a}) \leq \mathbb{T}G(\widehat{a})$, the inequality $a'_G(\widehat{a}) \leq a^*$ must hold.

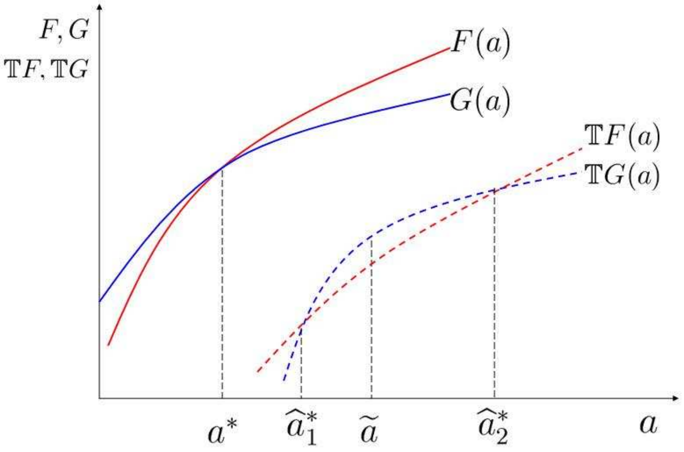

Figure 11: Illustration to the proof of Lemma 1

Statement (i) of Lemma 1 is also proven by contradiction. Suppose that $\mathbb{T}F(a)$ and $\mathbb{T}G(a)$ have multiple intersections, labeled by $\widehat{a}_1^*$ and $\widehat{a}_2^*$ on Figure 11, with $\widehat{a}_1^* < \widehat{a}_2^*$. Then there exists $\widetilde{a} \in (\widehat{a}_1^*, \widehat{a}_2^*)$ such that $\mathbb{T}F'(\widetilde{a}) = \mathbb{T}G'(\widetilde{a})$ and $\mathbb{T}F(\widetilde{a}) < \mathbb{T}G(\widetilde{a})$ (it is straightforward to verify that $\mathbb{T}F(\widetilde{a}) > \mathbb{T}G(\widetilde{a})$ for sufficiently lare a). On the one hand, $\mathbb{T}F'(\widetilde{a}) = \mathbb{T}G'(\widetilde{a})$ implies that the agents maximizing $\mathbb{T}F(\widetilde{a})$ and $\mathbb{T}G(\widetilde{a})$ derive the same current period utility. On the other hand, since $\widetilde{a}$ is in between the two intersection points, the properties of the policy functions $a'_F(a)$ and $a'_G(a)$ established above can be used to illustrate that $F(a'_F(\widetilde{a})) > G(a'_G(\widetilde{a}))$, which leads to an apparent contradiction (namely, it implies that $\mathbb{T}F(\widetilde{a}) < \mathbb{T}G(\widetilde{a})$ cannot hold). To see that $F(a'_F(\widetilde{a})) > G(a'_G(\widetilde{a}))$ indeed holds, observe that $a'_F(a) \geq a^*$ for all $a \geq \widehat{a}_1^*$ (since this is true at $\widehat{a}_1^*$ and, by concavity of $F(a)$, $a'_F(a)$ is monotone) and that $a'_G(a) \leq a^*$ for all $a \leq \widehat{a}_2^*$ (for the same reason). Thus $F(a'_F(\widetilde{a})) > F(a^*) = G(a^*) > G(a'_G(\widetilde{a}))$, which completes the proof of (i).

Finally, to verify (iii), notice that since $\widehat{a}_L$ and $\widehat{a}_H$ are the tangent points of $\mathbb{T}F(a)$ and $\mathbb{T}G(a)$ with their common tangent line and both are above $\underline{a}$, the following identities must hold:

$$\mathbb{T}G'(\widehat{a}_L) = \mathbb{T}F'(\widehat{a}_H) = \frac{\mathbb{T}F(\widehat{a}_H) - \mathbb{T}F(\widehat{a}_L)}{\widehat{a}_H - \widehat{a}_L}. \tag{12}$$

By the envelope condition, the first equality in (12) implies that the levels of current period consumption maximizing $\mathbb{T}G$ at $\widehat{a}_L$ and $\mathbb{T}F$ at $\widehat{a}_H$ are equal, and hence

$$\begin{aligned} &\mathbb{T}F(\widehat{a}_H) - \mathbb{T}F(\widehat{a}_L) = \beta\left(F(a'_F(\widehat{a}_H)) - G(a'_G(\widehat{a}_L)\right) \qquad \text{and} \\ &\widehat{a}_H - \widehat{a}_L = \frac{a'_F(\widehat{a}_H) - a'_G(\widehat{a}_L)}{1+r}. \end{aligned} \tag{13}$$

At the same time, interiority of $a'_F(\widehat{a}_H)$ and $a'_G(\widehat{a}_L)$ implies that, by the first order and envelope conditions,

$$\mathbb{T}G'(\widehat{a}_L) = \beta(1+r)G'(a'_G(\widehat{a}_L)) \text{ and } \mathbb{T}F'(\widehat{a}_H) = \beta(1+r)F'(a'_F(\widehat{a}_H)). \tag{14}$$

Combining (12)-(14), we obtain

$$G'(a'_G(\widehat{a}_L)) = F'(a'_F(\widehat{a}_H)) = \frac{F(a'_F(\widehat{a}_H)) - G(a'_G(\widehat{a}_L))}{a'_F(\widehat{a}_H) - a'_G(\widehat{a}_L)}. \tag{15}$$

The above identities can only be satisfied if $a'_F(\widehat{a}_H)$ and $a'_G(\widehat{a}_L)$ are the two wealth levels at which $F(a)$ and $G(a)$ are tangent to their common tangent line, i.e. $a'_F(\widehat{a}_H) = a_H$ and $a'_G(\widehat{a}_L) = a_L$. This completes the proof of Lemma 1. ■

Proof of Proposition 1:
Denote by a^*_t the wealth level at which $V_t(a)$ and $V_{t-1}(a)$ intersect; and by a^t_∞ the wealth level at which $V_t(a)$ and $V_\infty(a)$ intersect (conditions (a) and (b) of the Proposition imply that, by (i) of Lemma 1, each pair has at most one intersection). Also denote by $a'_t(a)$ the optimal savings policy of the agent maximizing $V_t(a)$ (solving (7)). Below only part (i) of the proposition is proven, the proof of part (ii) is symmetric. The proof is constructed in four steps; the first three establish the properties of $V(a)$, and the last – the properties of the consumer's optimal savings policy.

Step 1: Show that $V_0(a) > V_\infty(a)$ for large a implies that $V_t(a) \geq V_\infty(a)$ and $V_t(a) \leq V_{t-1}(a)$ for any t for sufficiently large a.

The first inequality follows by induction, since $V_{t+1} = \mathbb{T}V_t$ and $V_\infty = \mathbb{T}V_\infty$: if $V_t(a) \geq V_\infty(a)$ for large a (which, by assumption, holds for $t = 0$) then for those a (sufficiently large), for which $a'_\infty(a) > a^t_\infty$, $\mathbb{T}V_t(a) \geq \mathbb{T}V_\infty(a)$ must hold, since $a'_\infty(a)$ is a feasible saving policy for the agent maximizing $\mathbb{T}V_t(a)$.

The second inequality similarly follows by induction. First, notice that $V_1(a) \leq V_0(a)$ must hold for sufficiently large a: if $V_1(a) \geq V_0(a)$ for large a (this is the opposite to the supposition since, by (a), $V_0(a)$ and $V_1(a)$ have a unique intersection) then, by the above inductive argument, $V_t(a) \geq V_{t-1}(a)$ for sufficiently large a for any $t \geq 1$, which would imply that the sequence of value functions $\{V_t\}_{t=1}^\infty$ cannot converge to V_∞ at large a, since $V_t(a) \geq V_0(a) > V_\infty(a)$ for large a. Then, applying the same inductive argument once again, $V_1(a) \leq V_0(a)$ for large a yields $V_t(a) \leq V_{t-1}(a)$ for sufficiently large a for any $t \geq 1$.

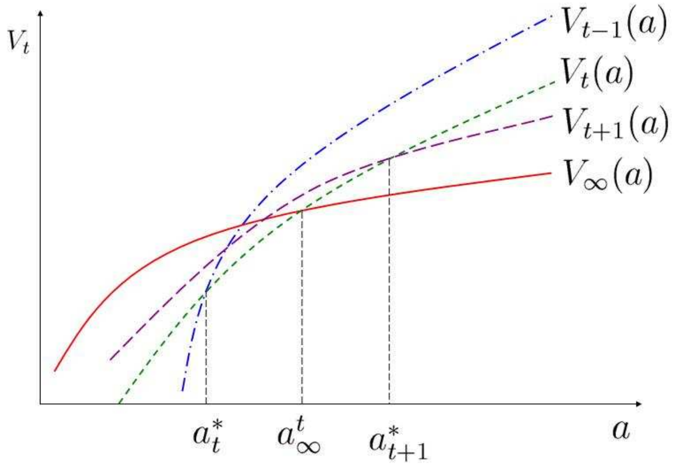

Figure 12: Illustration to Step 2 of the proof of Proposition 1

Step 2: Show that if $V_t(a) \leq \max\{V_\infty(a), V_{t-1}(a)\}$ for all $a \geq \underline{a}$ then $V_{t+1}(a) \leq \max\{V_\infty(a), V_t(a)\}$ for all $a \geq \underline{a}$.
This would imply that if $V_t(a)$ is irrelevant then $V_{t+k}(a)$, for $k \geq 1$, is also irrelevant. Correspondingly, if T is the smallest t for which $V_{t+1}(a) \leq \max\{V_\infty(a), V_0(a), V_1(a), ..., V_t(a)\}$ for all $a \geq \underline{a}$, then, for any $k \geq 1$, $V_{T+k}(a) \leq \max\{V_\infty(a), V_0(a), V_1(a), ..., V_T(a)\}$ for all $a \geq \underline{a}$ and hence $V(a) = \max\{V_\infty(a), V_0(a), V_1(a), ..., V_T(a)\}$.

The proof of this Step is by contradiction. Suppose that there exists $t \geq 1$ such that $V_t(a) \leq \max\{V_\infty(a), V_{t-1}(a)\}$ for all $a \geq \underline{a}$ but $V_{t+1}(a) > \max\{V_\infty(a), V_t(a)\}$ for some a. This implies that $a_{t+1}^* > a_\infty^t \geq a_t^*$ (see Figure 12).

First, notice that $a'_{t+1}(a_{t+1}^*) \geq a_{t+1}^*$ must hold. If the opposite were true then the sequence of the value functions $\{V_t(a)\}_{t=0}^{+\infty}$ would not be converging to $V_\infty(a)$. To see this, notice that $a'_{t+1}(a_{t+1}^*) < a_{t+1}^*$ implies that $V_{t+1}(a'_{t+1}(a_{t+1}^*)) > V_t(a'_{t+1}(a_{t+1}^*))$ and thus $V_{t+2}(a_{t+1}^*) > V_{t+1}(a_{t+1}^*)$. Since $V_{t+2}(a)$ and $V_{t+1}(a)$ have unique intersection and, by Step 1, $V_{t+1}(a) > V_{t+2}(a)$ for sufficiently large a, $V_{t+2}(a) > V_{t+1}(a) \geq V_t(a)$ for all $a \leq a_{t+1}^*$. By induction, it follows that $V_{t+k}(a) \geq V_{t+1}(a)$ for all $a \leq a_{t+1}^*$ and $k \geq 2$, implying that $\lim_{k\to+\infty} V_{t+k}(a_{t+1}^*) \geq V_{t+1}(a_{t+1}^*) > V_\infty(a_{t+1}^*)$ (the latter inequality holds strictly since, by supposition, $a_{t+1}^* > a_\infty^t$), which leads to a contradiction.

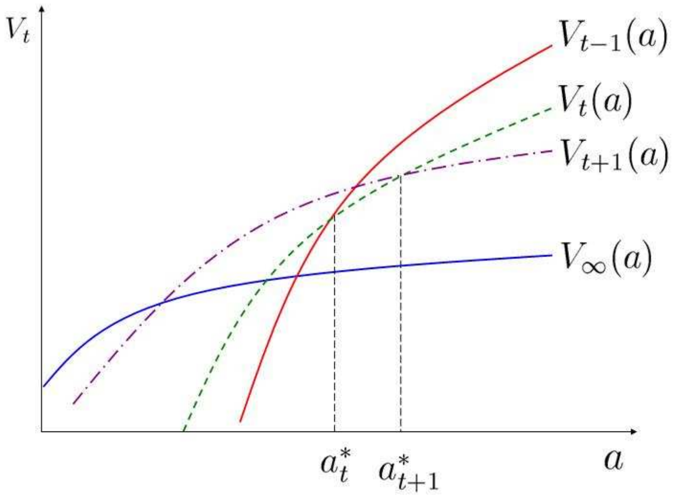

Figure 13: Illustration to Step 3 of the proof of Proposition 1

Second, $a'_{t+1}(a^*_{t+1}) \geq a^*_{t+1}$, together with $a^*_{t+1} > a^*_t$ (by supposition), implies that

$$\begin{aligned} V_{t+1}(a^*_{t+1}) &= u\left(a^*_{t+1} + y - h^* - \frac{a'_{t+1}(a^*_{t+1})}{1+r}, h^*\right) + \beta V_t(a'_{t+1}(a^*_{t+1})) \\ &< u\left(a^*_{t+1} + y - h^* - \frac{a'_{t+1}(a^*_{t+1})}{1+r}, h^*\right) + \beta V_{t-1}(a'_{t+1}(a^*_{t+1})) \leq V_t(a^*_{t+1}), \end{aligned}$$

which contradicts the definition of a^*_{t+1} (namely, that $V_{t+1}(a^*_{t+1}) = V_t(a^*_{t+1})$).

Hence, $V_{t+1}(a) \leq \max\{V_\infty(a), V_t(a)\}$ must hold for all $a \geq \underline{a}$.

Step 3: Verify that $a^*_{t+1} \leq a^*_t$ for all $t \leq T-1$ as long as $V_t(a) > \max\{V_\infty(a), V_0(a), ..., V_{t-1}(a)\}$ for some a.

Suppose that the opposite is true and $a^*_{t+1} > a^*_t$ for some $t \leq T-1$ (see Figure 13). Intuitively, this cannot be dynamically consistent because this would imply that there exist wealth levels at which the agent strictly prefers to wait for $t+1$ periods before making the adjustment, but there are no wealth levels at which he wants to wait for t periods. The supposition implies that, by (ii) of Lemma 1, $a'_{t+1}(a^*_{t+1}) \leq a^*_t$. Correspondingly, $V_{t+1}(a'_{t+1}(a^*_{t+1})) > V_t(a'_{t+1}(a^*_{t+1}))$, and thus $V_{t+2}(a^*_{t+1}) > V_{t+2}(a^*_{t+1})$ (since $a'_{t+1}(a^*_{t+1})$ is a feasible saving policy for the agent at a^*_{t+1} maximizing $V_{t+2}(a)$). Since $V_{t+2}(a)$ and $V_{t+1}(a)$ have at most one intersection and, by Step 1, $V_{t+2}(a) < V_{t+1}(a)$ for sufficiently large a, it follows that $V_{t+2}(a) > V_{t+1}(a) \geq V_t(a)$ for all $a \leq a^*_{t+1}$. Inductive application of this argument yields $V_{t+k}(a^*_{t+1}) > V_{t+1}(a^*_{t+1})$ for any $k \geq 2$. Since $V_{t+1}(a^*_{t+1}) > V_\infty(a^*_{t+1})$ (since

$V_t(a) > \max\{V_\infty(a), V_0(a), ..., V_{t-1}(a)\})$, it follows that $\lim_{k\to+\infty} V_{t+k}(a^*_{t+1}) > V_\infty(a^*_{t+1})$, which is an obvious contradiction.

Step 4: Verify that $a'(a) \in (a^*_t, a^*_{t-1}]$ for all $a \in (a^*_{t+1}, a^*_t]$. Intuitively this must hold because it says the agent's behavior is dynamically consistent: if the current wealth is such that the agent chooses to wait for t periods before making the adjustment then the agent's next period's wealth must be such that he would plan to wait for exactly $t-1$ periods.

Formally, when $a \in (a^*_{t+1}, a^*_t]$, $V(a) = V_t(a)$ and thus $a'(a) = a'_t(a)$. Since $V_t(a)$ is strictly concave, $a'_t(a)$ is strictly increasing, and hence we need to verify that $a'_t(a^*_t) \leq a^*_{t-1}$ and $a'_t(a^*_{t+1}) \geq a^*_t$. Both inequalities directly follow from (ii) of Lemma 1; the first one holds since $V_t = \mathbb{T}V_{t-1}$ and $V_{t-1} = \mathbb{T}V_{t-2}$ (i.e. V_{t-1} and V_{t-1} play the roles of F and G in the statement of Lemma 1 respectively), and the second one holds since $V_{t+1} = \mathbb{T}V_t$ and $V_t = \mathbb{T}V_{t-1}$ (i.e. V_t and V_{t-1} play the role of F and G here).

■

Proof of Proposition 2:

The proof of all value functions from the sequence $\{V_t\}_{t=0}^{\infty}$ having a single common tangent line is provided in the main text. To verify that $V_\infty(a)$ is tangent to the same line at a^*, it is crucial to notice that $\beta(1+r) = 1$ implies that the wealth level of the agent staying with his initial option forever does not change over time: $a'_\infty(a) = a$.

If $\max\{V_\infty(a), V_0(a)\}$ is globally concave and $V_\infty(a)$ and $V_0(a)$ intersect at some a^* then it is easy to see that $V_1(a^*) = V_0(a^*) = V_\infty(a^*)$ as well as $V'_1(a^*) = V'_0(a^*) = V'_\infty(a^*)$. By induction, the same properties at a^* hold for all value functions from the sequence $\{V_t\}_{t=1}^{\infty}$, implying that $V(a) = \max\{V_\infty(a), V_0(a)\}$ and the statement of Proposition 2 trivially applies.

Now suppose that $\max\{V_\infty(a), V_0(a)\}$ is not globally concave. First, note that this implies that $\max\{V_\infty(a), V_t(a)\}$ cannot be globally concave for any t: if $V'_\infty(a^t_\infty) = V'_t(a^t_\infty)$ at a^t_∞ at which $V_\infty(a^t_\infty) = V_t(a^t_\infty)$, the first order and envelope conditions would imply that $V_\infty(a^t_\infty) = V_{t-1}(a^t_\infty)$ and $V'_\infty(a^t_\infty) = V'_{t-1}(a^t_\infty)$ must hold, and, by induction, the same applies for all V_k, $k \in \overline{0,t}$, including $V_0(a)$ (which contradicts to $\max\{V_\infty(a), V_0(a)\}$ not being globally concave). Second, the fact that $V'_\infty(a^t_\infty) < V'_t(a^t_\infty)$ for every t (when case (i) of Proposition 1 applies) implies that the saving policy $a_\infty(a)$ of the agent maximizing $V_\infty(a)$ is strictly suboptimal for the agent with the current asset level a_∞^t maximizing $V_t(a)$, and hence

$$\begin{aligned} V_{t+1}(a_\infty^t) &> u(a^* + y - h_0 - \frac{a'_\infty(a_\infty^t)}{1+r}, h_0) + \beta V_t(a'_\infty(a_\infty^t)) \\ &= u(a_\infty^t + y - h_0 - \frac{a_\infty^t}{1+r}, h_0) + \beta V_t(a_\infty^t) \\ &= u(a_\infty^t + y - h_0 - \frac{a_\infty^t}{1+r}, h_0) + \beta V_\infty(a_\infty^t) = V_\infty(a_\infty^t). \end{aligned}$$

Here the second row reflects that $a'_\infty(a) = a$ under $\beta(1+r) = 1$, and the third row applies the definition of a_∞^t. This implies that $V_{t+1}(a) > \max\{V_\infty(a), V_0(a), ..., V_t(a)\}$ for some a for any $t \geq 0$, and hence the sequence of the cutoffs a_t^* is infinite (i.e. $T = \infty$). Since $\lim_{t\to\infty} V_t(a) = V_\infty(a)$, it trivially follows that $\lim_{t\to\infty} a_\infty^t = \lim_{t\to\infty} a_t^* = a^*$ and that $V_\infty(a)$ is tangent to the common tangent line of $\{V_t\}_{t=0}^\infty$ at exactly a^*.

All the above arguments also apply when case (ii) of Proposition 1 holds, thus Proposition 2 holds independently of whether the agents exhibiting transitory behavior are transiting upwards or downwards.

■

Proof of Proposition 3:

Consider two restricted problems:

$$V_0(a; h^* \geq h_0) = \max_{a', h^* \geq h_0} \{u(a + y - h^* - \eta - \frac{a'}{1+r}, h^*) + \beta V^*(a'; h^*)\} \tag{16}$$

and

$$V_0(a; h^* \leq h_0) = \max_{a', h^* \leq h_0} \{u(a + y - h^* - \eta - \frac{a'}{1+r}, h^*) + \beta V^*(a'; h^*)\}, \tag{17}$$

where $V^*(a; h^*)$ s the value of staying in house h forever defined in (4). We need to verify that $V_0(a; h^* \geq h_0)$ and $V_\infty(a)$, as well as $V_0(a; h^* \leq h_0)$ and $V_\infty(a)$, satisfy conditions (a) and (b) of Proposition 1. Once these properties are established, the rest of the proof is straightforward (and was outlined in the main text, including footnote 36).

(a) Single crossing of $V_0(a; h^* \geq h_0)$ and $V_\infty(a)$:

Clearly, $V_0(a; h^* \geq h_0) > V_\infty(a)$ for sufficiently large a and $V_0(a; h^* \geq h_0) = V_\infty(a - \eta) < V_\infty(a)$ for sufficiently small a, i.e. $V_0(a; h^* \geq h_0)$ and $V_\infty(a)$ cross at least once.

Since $\beta(1+r) = 1$ and there are no borrowing constraints, the agents solving (8) and

(16) choose constant over times profiles of consumption and housing. Hence,

$$V_\infty(a_0) = \frac{1}{1-\beta} u\left(\frac{r}{1+r}a_0 + y - h_0, h_0\right) \tag{18}$$

and

$$V_0(a_0; h^* \geq h_0) = \frac{1}{1-\beta} u\left(\frac{r}{1+r}a_0 + y - h^* - \frac{r}{1+r}\eta, h^*\right), \tag{19}$$

where h^* is the optimally chosen level of housing (found from the first order condition $u_1\left(\frac{r}{1+r}a_0 + y - h^* - \frac{r}{1+r}\eta, h^*\right) = u_2\left(\frac{r}{1+r}a_0 + y - h^* - \frac{r}{1+r}\eta, h^*\right)$). Therefore,

$$\begin{aligned}(1-\beta)\frac{1+r}{r}V_0'(a_0; h^* \geq h_0) &= u_1\left(\frac{r}{1+r}a_0 + y - h^* - \frac{r}{1+r}\eta, h^*\right)\\ &\geq u_1\left(\frac{r}{1+r}a_0 + y - h^* - \frac{r}{1+r}\eta, h_0\right)\\ &> u_1\left(\frac{r}{1+r}a_0 + y - h_0, h_0\right) = (1-\beta)\frac{1+r}{r}V_\infty'(a_0),\end{aligned} \tag{20}$$

where the first inequality is a consequence of weak supermodularity of $u(c,h)$ and the second inequality holds since $u(c,h)$ is concave in c, $h^* \geq h_0$ and hence $h^* + \frac{r}{1+r}\eta > h_0$. Thus $V_0'(a_0; h^* \geq h_0) > V_\infty'(a_0)$ for all a_0, implying that $V_0(a; h^* \geq h_0)$ and $V_\infty(a)$ can have at most one intersection and hence satisfy condition (b) of Proposition 1.

Single crossing of $V_0(a; h^* \leq h_0)$ and $V_\infty(a)$:

Obviously, $V_0(a; h^* \leq h_0) > V_\infty(a)$ for sufficiently small a and $V_0(a; h^* \geq h_0) = V_\infty(a - \eta) < V_\infty(a)$ for sufficiently large a, i.e. $V_0(a; h^* \leq h_0)$ and $V_\infty(a)$ cross at least once.

Note that the approach used above cannot be used here since $h^* \leq h_0$ implies that $h^* + \frac{r}{1+r}\eta > h_0$ may be violated. Thus I take a different route and show that $V_\infty'(a_0) \leq V_0'(a_0; h^* \geq h_0)$ implies that $V_\infty(a_0) \geq V_0(a_0; h^* \geq h_0)$ for all a_0 (which guarantees single crossing). Employing the envelope conditions and weak supermodularity of

$u(c, h)$ (in conjunction with $h^* \leq h_0$), we obtain

$$\begin{aligned}
&u_1\left(\frac{r}{1+r}a_0 + y - h_0, h_0\right) = (1-\beta)\frac{1+r}{r}V'_\infty(a_0) \\
&\leq (1-\beta)\frac{1+r}{r}V'_0(a_0; h^* \geq h_0) = u_1\left(\frac{r}{1+r}a_0 + y - h^* - \frac{r}{1+r}\eta, h^*\right) \\
&\leq u_1\left(\frac{r}{1+r}a_0 + y - h^* - \frac{r}{1+r}\eta, h_0\right).
\end{aligned}$$

The inequality between the first and the last terms above, together with concavity of $u(c, h)$ in c implies that

$$\frac{r}{1+r}a_0 + y - h_0 \geq \frac{r}{1+r}a_0 + y - h^* - \frac{r}{1+r}\eta,$$

and hence, since $h_0 \geq h^*$, $V_\infty(a_0) \geq V_0(a_0; h^* \geq h_0)$. Thus $V_0(a; h^* \leq h_0)$ and $V_\infty(a)$ satisfy (b) of Proposition 1.

(b) Single crossing of $V_0(a; h^* \geq h_0)$ and $\mathbb{T}V_0(a; h^* \geq h_0)$ is easiest to establish using the recursive representation of both functions. As above, it is enough to show that $V'_0(a) = \mathbb{T}V'_0(a)$ implies that $V_0(a) \geq \mathbb{T}V_0(a)$. For brevity, I drop the reference to $h^* \geq h_0$ in the notation for $V_0(a; h^* \geq h_0)$.

Suppose that $V'_0(a) = \mathbb{T}V'_0(a)$ at some $a > \underline{a}$. Then from the envelope conditions, supermodularity of $u(c, h)$, $h^* \geq h_0$ and concavity of $u(c, h)$ it follows that $u(a + y - \eta - h^* - \frac{a'_0}{1+r}, h^*) \geq u(a + y - h_0 - \frac{a'_1}{1+r}, h_0)$ (where a'_0 and a'_1 are the optimal saving levels maximizing $V_0(a)$ and $TV_0(a)$ respectively). Thus the agent maximizing $V_0(a)$ enjoys higher current period utility than the agent maximizing $\mathbb{T}V_0(a)$. It remains to show that the same relationship holds for the corresponding continuation values $V^*(a'_0; h^*)$ and $V_0(a'_1)$.

The combination of the envelope and first order conditions implies that $\mathbb{T}V'_0(a) = \beta(1+r)V'_0(a'_1)$. Since $\beta(1+r) = 1$ and $V'_0(a) = \mathbb{T}V'_0(a)$, it follows that $V'_0(a) = V'_0(a'_1)$, and hence $a'_1 = a$ and $V_0(a'_1) = V_0(a)$. From the maximization problem for $V_0(a)$, it follows that $V'_0(a) = \beta(1+r)V_1^{*}(a'_0; h^*)$. Since $\beta(1+r) = 1$ and $V_0(a) = \max_{h^* \geq h_0}\{V^*(a - \eta; h^*)\}$, it follows that $V_1^*(a'_0; h^*) = V_0(a)$. Therefore, in both decision problems, for $\mathbb{T}V_0(a)$ and $V_0(a)$, $V'_0(a) = \mathbb{T}V'_0(a)$ also leads to the equality of the continuation values. Hence, $V'_0(a) = \mathbb{T}V'_0(a)$ implies that $V_0(a) \geq \mathbb{T}V_0(a)$, which proves that $V_0(a; h^* \geq h_0)$ indeed satisfies condition (a) of Proposition 1.

Single crossing of $V_0(a; h^* \leq h_0)$ and $\mathbb{T}V_0(a; h^* \geq h_0)$ is established using a symmetric argument.

Therefore, Proposition 1 can be separately applied to the two restricted problems, switching to a bigger house and switching to a smaller house, which creates non-empty unique upward and downward transitory intervals. Footnote 36 proves that these transitory intervals are disjoint, namely that $a_L^* < a_H^*$. This completes the proof of Proposition 2. ■